A Model of Positive Self-Image in Subjective Assessments

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This paper suggests a mechanism that describes individuals' positive self-image in subjective assessments of their relative abilities. The mechanism assumes individuals have heterogeneous production functions that determine ability as a function of multiple skills; make skill-enhancing investments with the goal of maximizing their ability; and make ability comparisons using their own production function. Within this framework, the paper provides conditions under which there is positive self-image. Positive self-image is increasing in the ease of the task, the number of different skills needed for the task, and the variability of production technologies in the population. (*JEL* A12, D01)

Many have noted that people tend to have overly positive assessments of their relative abilities. Adam Smith (1776) wrote that "the over-weening conceit which the greater part of men have of their own abilities, is an ancient evil remarked by the philosophers and moralists of all ages." Contemporary psychologists agree that "on nearly any dimension that is both *subjective* and *socially desirable,* most people see themselves as better than average."[1]

Positive self-image may influence behavior in economically relevant situations. Smith suggests that people's overly positive view of their own abilities explains gambling behavior and the decision of individuals to become soldiers. Myers points out that merit pay may lead to low morale when 90 percent or more of employees rates themselves as above average.[2]

We argue that much of the evidence about positive self-image fits comfortably in the standard model that we describe. In our model, ability is a function of a vector of skills. There is a subjective component to the definition of ability, in that different individuals can hold different opinions about how skills combine to determine an ability level. We capture this ambiguity by assuming that ability is an increasing function of skills, but that different agents use different functions to evaluate ability. The fact that we permit more than one measure of ability is what makes our comparisons subjective. Individuals begin with an initial endowment and make an investment to improve their ability. We assume that when an individual responds to questions about relative standing, he responds egocentrically: he uses his own production function to compare his final skills to those of others in the population.

As an example, driving is an ability that depends on several individual skills: knowledge of laws; ability to merge into freeway traffic; parallel parking; controlling a vehicle on icy roads; and so on. Different individuals disagree about how much each of these skills contributes to good driving. Parking skill is a significant part of ability for the urban driver. For someone in a cold climate, how well he can control a car in a snow storm is an important factor in driving

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[1] The quotation is from David Myers (1996, p. 54), a textbook on social psychology. The book presents several examples of positive self-image.

[2] Myers (1996, p. 63) cites studies by Patricia K. Cross (1977) of the relative self-image of college professors.

ability. These differences will lead different drivers to augment their skills in different ways.

Our measure of an individual's self-image is a number between zero and one. This number is equal to the fraction of the population that, in the individual's opinion, has a lower ability level. A strong form of positive self-image arises when the fraction of individuals who view themselves to be in the bottom p percent of the population is less than p for all p. In this case, more than 10 percent of individuals in the population view their ability as greater than 90 percent of others in the population; half of the population would claim to be above the median; and so on.

Our basic model generates this kind of positive self-image. Without the ability to add to skills, the population would typically be well calibrated: precisely p percent of the population would claim to be better than $1 - p$ percent of the others. Positive self-image arises because individuals tailor their skill augmentation to their own production technology. The driver who values parking will work to improve her parking skills and ultimately rate her ability more highly than that of someone with an identical initial endowment who chooses to improve his freeway driving.[3]

We are convinced by the evidence that positive self-image is real, although our results do suggest conditions that will lessen or reverse its appearance. Nonetheless, we argue that there is a parsimonious way to organize the findings that does not depend on assuming that individuals process information irrationally or that self esteem enters directly into their utility functions.

The most closely related paper is by Eric Van den Steen (2004). In Van den Steen's basic model, an agent must choose from a finite number of actions. Agents have heterogeneous beliefs about the probability that a given action succeeds. Since each agent selects the action that (in his view) is most likely to succeed, each agent believes that his choice is at least as good as the choices made by others in the population, and everyone believes that the other agents overestimate their probability of succeeding. While we formulate our model in terms of heterogeneous technologies, formally Van den Steen's basic model is a limit case of the model we use in most of the paper.

Van den Steen's model contains the two basic elements of our approach. Individuals use different criteria to evaluate their decisions, and they make choices. Given the similarity of the basic structure, it is not surprising that Van den Steen's paper contains versions of several of the results in this paper. In addition to demonstrating the existence of excessive optimism, he demonstrates that excessive optimism increases with the riskiness of the distribution of beliefs (comparable to Proposition 3) and with number of projects (comparable to Proposition 4). In our model, individuals have different technologies and different endowments. Adding the possibility of different endowments allows us to state propositions that are not available in Van den Steen's framework. For example, no individual can have negative self-image in Van den Steen's model. Our results on the sensitivity of self-image to the distribution of income (Propositions 6 and 9) have no counterpart in Van den Steen's paper. Sections V, VI, and VII also have no parallels in Van den Steen's work.

The next section of the paper introduces the model. Section II states a general proposition that gives sufficient conditions for positive self-image. A symmetric result identifies sufficient conditions for negative self-image. Proposition 2 shows that positive self-image arises when individuals invest in skills to maximize their ability. The result not only guarantees that at least half of the population views itself to be above the median, but also extends the "above-median" effect to all percentiles. Section III solves our model for a special family of technologies that we use for subsequent comparative-statics analysis. Section IV contains results on how positive self-image is influenced by the variability of production technologies in the population. Informally, we show that the amount of positive self-image in the population grows with the ambiguity of the ability being assessed. Section V gives conditions under which positive self-image is more pronounced for easy tasks than for hard ones. Section VI investigates the finding that people tend to claim that important skills are the ones that they possess in largest quantities. Section VII models the tendency for positive self-image to be more pronounced in individuals with lower objective skill levels. Section VIII suggests some implications of our model that have not, to our knowledge, been subject to detailed empirical

[3] Ola Svenson (1981) identifies positive self-image in drivers.

investigation. In particular, we discuss environments that would lead to negative self-image. Section IX reviews some related research. Section X concludes the paper. The Appendix contains the proofs of propositions.

I. The General Model

There is a large population of individuals. Individuals are characterized by a vector of final skill levels ($k \in \mathbb{R}^n_+$) and a parameter $\lambda \in \mathbb{R}^n_+$. There is a continuous function, called the technology, $T(\cdot)$, which transforms skill levels into a real-valued ability. $T(k; \lambda)$ is the ability of an individual with final skill vector k if the technological parameter is λ. We assume that there is a probability measure μ defined on Borel subsets of a compact subset C of $\mathbb{R}^n_+ \times \mathbb{R}^n_+$ and interpret $\mu(X)$ as the fraction of the population with $(k, \lambda) \in X$. We denote by $\mu(S|\lambda)$ the conditional probability of the set $S \subset \mathbb{R}^n_+$ of final skill levels when technology is fixed at λ.

The paper investigates how individuals in the population rank themselves relative to other members of the population. We next introduce notation that allows us to express answers to the question: "What fraction of the population has ability greater than yours?"

An individual with final skill level k and technology λ perceives that he has ability at least as great as anyone with final skills in the set $K(k, \lambda)$:[4]

$$(1) \qquad K(k, \lambda) = \{k' : T(k, \lambda) \geq T(k', \lambda)\}.$$

We define an individual's *self-image* as the fraction of the population that has a lower ability:

$$(2) \qquad SI(k, \lambda) = \mu\{(k', \lambda') : k' \in K(k, \lambda)\}.$$

Our definition of self-image is subjective because different individuals evaluate final skills using different technologies. It is egocentric because each individual evaluates ability using his own technology.

Results from social psychology justify our modeling approach. David Dunning et al. (1991) and Dunning and Andrew F. Hayes (1996) are representative of papers that demonstrate that individuals make egocentric comparisons when evaluating the abilities of others. That is, in order to evaluate the behavior of others, they apply the standards that they use on themselves. The population would exhibit a "better than median effect" if $SI(k, \lambda) \geq 1/2$ for more than half of the population. Our notion of positive self-image is more stringent. We require that the fraction of the population that perceives itself to have ability levels in the top p of the population is greater than p for all $p \in (0, 1)$. Let

$$(3) \qquad B(p) = \{(k, \lambda) : SI(k; \lambda) \geq 1 - p\}$$

so that $B(p)$ is the set of individuals who perceive that their skills are in the top p-cile of the population. A population exhibits *positive self-image* if

$$(4) \qquad \mu(B(p)) \geq p \text{ for all } p \in (0, 1).$$

Positive self-image is strict if the inequality in (4) is strict for all $p \in (0, 1)$.

Similarly, let $K^-(k, \lambda) = \{k' : T(k, \lambda) \leq T(k', \lambda)\}$; $SI^-(k, \lambda) = \mu\{(k', \lambda') : k' \in K^-(k, \lambda)\}$; and $B^-(p) = \{(k, \lambda) : SI^-(k; \lambda) \geq 1 - p\}$. There is *negative self-image* if $\mu(B^-(p)) \geq p$ for all $p \in [0, 1]$.[5]

If final skills are distributed independently of λ, then typically the population will exhibit neither positive nor negative self-image.[6] The key to our analysis is that we assume that individuals have an initial endowment I and then choose their skills (subject to constraints) to maximize their final ability level. As a result, final skills will be correlated with technologies. For most of the paper, I will be a real variable that we call income. More generally, I could represent an initial vector of skills and a budget that could be used to add to these skills. Our basic result holds under more general assump-

[4] The focus of this paper is on nondegenerate cases where $\mu(\{(k, \lambda) : T(k, \lambda) = c\}) = 0$ for all (k, λ). In these nondegenerate cases, our results would apply if the definition (1) used a strict inequality.

[5] If $\mu(\{(k, \lambda) : SI(k, \lambda) = 1 - p\}) = 0$, then the population exhibits negative self-image if $\mu(B(p)) \leq p$ for all $p \in [0, 1]$. The somewhat more involved definition in the text properly accounts for the possibility of ties—that a positive fraction of the population has identical final ability.

[6] The population may exhibit positive self-image in degenerate cases. For example, if everyone were identical in technology and initial skill level, then the definition implies that everyone ranks himself as best.

tions: for the results in Section II, we assume that I is an element of $\mathbb{R}_+^m$ for some m. Let $A(I) \subset \mathbb{R}_+^n$ be a nonempty, convex, and compact set of final skills that an individual with initial income I may acquire. Assume that if $I < I'$, then $A(I)$ is strictly contained in $A(I')$. If an individual has initial income I and technology parameter λ, then the optimization problem

$$\text{(5)} \qquad \max T(k; \lambda) \text{ subject to } k \in A(I)$$

determines his final skill level. Problem (5) has a solution since $A(I)$ is nonempty and compact and $T(\cdot)$ is continuous. Denote by $\phi(I, \lambda)$ a measurable selection from the solution correspondence of (5)[7] and $T^*(I, \lambda) = T(\phi(I, \lambda), \lambda)$ the value function. Assume that initial endowments and technologies are independently distributed and the measure μ_0 defined on subsets of $\mathbb{R}_+^m \times \mathbb{R}_+^n$ describes the distribution of initial skills and technologies. For $X \subset \mathbb{R}_+^m \times \mathbb{R}_+^n$ interpret $\mu_0(X)$ as the fraction of the population with initial characteristics (I, λ) in X. The distribution of initial incomes and technologies determines the distribution of final skills and technologies μ through the relationship:

$$\text{(6)} \quad \mu(X) = \mu_0\{(I, \lambda) : (\phi(I, \lambda), \lambda) \in X\}.$$

In the *skill acquisition model,* individuals are described by their initial income I and technology λ; I and λ are independently distributed; final skills are selected to solve (5); and (6) describes the probability distribution over the space of final skills and technologies.

Nontrivial positive self-image requires differences in technologies. We say that technologies are *distinct* if for each λ', $\phi(I, \lambda) \neq \phi(I, \lambda')$ almost everywhere. Technologies are distinct if people with different technologies solve problem (5) differently. In Section III we introduce a family of distinct technologies which we use for our comparative-statics exercises.

One property of the skill acquisition model is immediate from the definitions. It follows from problem (5) that

$$\text{(7)} \qquad T^*(I, \lambda) \geq T(\phi(I, \lambda'), \lambda).$$

Therefore, each individual perceives himself to have ability at least as great as anyone with less income. Consequently, if individuals differ only in their technologies, every individual believes his own skills are the best in the population. There is positive self-image in the strongest possible sense. This result holds in Van den Steen's (2004) model, where people have heterogeneous beliefs but identical investment opportunities.

There is another immediate implication of the definitions. Let

$$D^*(I, \lambda) = T^*(I, \lambda) - E_{(I', \lambda')}\{T(\phi(I', \lambda'); \lambda)\}$$

be the difference between an individual's ability and the expected ability of the population, where ability is measured according to that individual's technology. Consequently, $D^*(I, \lambda)$ is a subjective measure of the amount by which the (I, λ)-individual is above (or below) the average. We refer to $D^*(I, \lambda)$ as individual i's *ability gap.* The ability gap is increasing in I and can be negative for individuals who have low endowments. Since income is distributed independently of technology, it follows from (7) that the ability gap averaged over all of the technologies in the population (either conditional or unconditional on income) is nonnegative in the skill acquisition model. Some of our comparative-statics results identify conditions that increase the expected ability gap.

II. General Results

This section presents a formulation of the basic result on positive self-image when individual investments determine final skill levels. Our first result, however, provides sufficient conditions on the joint distribution of final skill levels and technologies that imply either positive or negative self-image.

In order to state the result, we define self-image relative to a technology.

$$\text{(8)} \qquad SI(k, \lambda | \lambda') = \mu(\{K(k, \lambda) | \lambda'\})$$

and

$$\text{(9)} \qquad SI^-(k, \lambda | \lambda') = \mu(\{K^-(k, \lambda) | \lambda'\}).$$

[7] None of our results depends on the way in which one selects optimizers. For most of the analysis, we assume that $T(k; \lambda)$ is strictly concave in k for all λ and that $A(\cdot)$ is convex, so the solution to problem (5) is unique. No insight is lost by assuming that the solution correspondence is single valued.

Definitions (8) and (9) evaluate an individual's standing relative to people in the population who have a common technology. In particular, $SI(k, \lambda|\lambda)$ is a measure of "in-group" self-image. There is (for nondegenerate cases) no positive or negative self-image relative to one's own group because individuals with the same technology agree on the ranking of final skill levels.[8]

PROPOSITION 1: *If*

$$SI(k, \lambda|\lambda') \geq SI(k, \lambda|\lambda) \text{ for all } k, \lambda, \lambda', \tag{10}$$

then the population exhibits positive self-image. If the inequality is strict for all $\lambda \neq \lambda'$, *then positive self-image is strict.*

If

$$SI^{-}(k, \lambda|\lambda') \geq SI^{-}(k, \lambda|\lambda) \text{ for all } k, \lambda, \lambda', \tag{11}$$

then the population exhibits negative self-image. If the inequality is strict for all $\lambda \neq \lambda'$, *then negative self-image is strict.*

Proposition 1 states that positive self-image arises if in-group self-image is lower than self-image relative to a group with a different technology. Recalling that $SI^{-}(k, \lambda)$ is the fraction of the population that a (k, λ)-individual perceives as superior, negative self-image arises if in-group self-image is higher than self-image relative to an external group.

The Appendix contains a proof of Proposition 1 (and all other results that require proof). The idea of the proof is simple. Self-image is an average of the self-image relative to all technologies: $SI(k, \lambda) = E_{\lambda'}\{SI(k, \lambda|\lambda')\}$. When an individual compares himself to people with the same technology, there is neither positive nor negative self-image. So if an individual perceives his relative position to be lower when compared to people with the same technology than when compared to people with different technologies (this is (10)), she will have positive self-image.

If k and λ are distributed independently, there will be no (strict) positive self-image. The next result demonstrates that when k is chosen to maximize $T(\cdot)$, k and λ will be correlated. Indeed, we expect to find $SI(k, \lambda|\lambda) < SI(k, \lambda|\lambda')$ for $\lambda \neq \lambda'$.

PROPOSITION 2: *In the skill acquisition model, the population exhibits positive self-image. Positive self-image is strict if technologies are distinct.*

Proposition 2 provides a natural setting in which positive self-image arises. Positive self-image can also arise if individuals selected their technology to match their skills (that is, if k is fixed but individuals select λ to maximize ability). This alternative explanation has been proposed by psychologists.

We have not thought up a model of skill acquisition that would give rise to (11). Negative self-image is a theoretical possibility within our model, but we do not have a realistic model of choice that generates negative self-image. Nevertheless, condition (11) is instructive because it states that negative self-image arises when there is a mismatch between final skill levels and technologies.

III. The CES Model

Our basic result on the existence of positive self-image holds for the skill acquisition model in which initial skills are distributed independently of technologies, and final skills are selected to maximize subjective ability. In order to state and prove propositions describing how different environments influence positive self-image, we analyze a special case of the model in which technologies have constant elasticity of substitution (CES). Assume that $\lambda \in \mathbb{R}^n_+$; $T(k;\lambda) = \sum_{i=1}^{n} \lambda_i k_i^{\rho}$ for $\rho \in (0, 1)$;[9] I is distributed on $[\underline{I}, \bar{I}]$ for $0 \leq \underline{I} < \bar{I}$; and $A(I) = \{k = (k_1, \ldots, k_n) \in \mathbb{R}^n_+ : \sum_{i=1}^{n} k_i = I\}$. In this formulation, the individual allocates his

[8] In-group positive (and negative) self-image is possible in degenerate situations. For example, if all individuals have the same final skill vector k, then $SI(k, \lambda|\lambda) = SI^{-}(k, \lambda|\lambda) = 1$.

[9] CES technologies are often written $T(k; \lambda)^{1/\rho}$. Positive self-image is an ordinal property, so results about positive self-image do not change when an increasing transformation—exponentiation—is applied. We omit the exponent $1/\rho$ to simplify notation.

endowment (hereafter, income) I to enhance his skills. Provided that $\rho < 1$, $T(\cdot)$ is strictly concave. In the limit case, $\rho = 1$, technologies are linear. We assume that I and λ are independently distributed and denote by $F(\cdot)$ the continuous cumulative distribution function on income. $F(I)$ is the fraction of the population that has income less than or equal to I.

When everyone has income $I = 1$, our CES model reduces to Van den Steen's model in the limit case $\rho = 1$: interpret the n skills as projects and λ_i as the probability that the i^{th} project succeeds. Individuals then invest in the project that they believe is most likely to succeed. What we call ability Van den Steen interprets as the probability of success.

For the CES model, we can solve (5) explicitly. Let $k^*(I, \lambda) = (k_1^*(I, \lambda), \ldots, k_n^*(I, \lambda))$ denote individual (I, λ)'s ability maximizing choice of final skills and let $\sigma = 1/(1 - \rho)$. Straightforward calculation verifies that

$$k_i^*(I, \lambda) = I \frac{\lambda_i^\sigma}{\sum_{j=1}^{n} \lambda_j^\sigma}. \tag{12}$$

An individual with technology λ will perceive that he is strictly more able than someone with the same income but a different technology. One can measure the extent of the difference by asking: What level of income I' would an individual with technology λ' need in order for the (I, λ)-type individual to view the (I', λ')-type as equally skilled? That is, what is the solution to

$$T^*(I, \lambda) = T(k^*(I', \lambda'), \lambda) \tag{13}$$

where $T^*(\cdot)$ is the optimal value function for an individual with income I and technology λ?

The solution to equation (13) takes the form $I' = Ih(\lambda'; \lambda)$ where

$$h(\lambda'; \lambda) = \frac{\left(\sum_{i=1}^{n} \lambda_i^\sigma\right)^{1/(\sigma-1)} \sum_{i=1}^{n} (\lambda_i')^\sigma}{\left(\sum_{i=1}^{n} \lambda_i (\lambda_i')^{\sigma-1}\right)^{\sigma/(\sigma-1)}}. \tag{14}$$

By construction, it must be that $h(\lambda'; \lambda) \geq 1$. Direct calculations also confirm that the inequality is strict whenever λ' is not a positive multiple of λ. In the CES model, the advantage of an individual with technology λ over individuals with technology λ' is equivalent to inflating his income by the factor $h(\lambda'; \lambda)$. The functional form of the $T^*(\cdot)$ leads $h(\cdot)$ to be independent of I.

These computations allow us to confirm Proposition 2 directly for the CES model. It follows from the definition of $h(\cdot)$ that

$$SI(k, \lambda | \lambda') = F(Ih(\lambda'; \lambda)). \tag{15}$$

Equation (15) states that an individual with technology λ perceives his ability to be as great as that of individuals with higher income and a different technology.

IV. Positive Self-Image and Ambiguity

The most critical feature leading to positive self-image is how easy it is for individuals to apply egocentric interpretations of the ability under study. Empirical studies can gain insight into how the definition of ability influences self appraisals by controlling the number of skills that subjects can count as relevant to an ability. For example, Dunning et al. (1989) manipulated the number of attributes that subjects could use to describe a specific trait. Some subjects were given a list of six potential attributes to construct their trait definitions, others were given two or four attributes, and a control group was given none. They found that the more restrictive the menu of attributes, the lower were subjects' above-median effects. Richard B. Felson (1981) and Dunning et al. (1989) compare subjects' ratings on different abilities and conclude that the more ambiguous the trait, the greater the evidence of positive self-image.[10] Scott T. Allison et al. (1989, p. 277) summarize the relevant literature by stating that "the less ambiguous, private, or subjective the attribute is, the less subject it is to self-serving exaggeration."

In this section, we propose several ways to study the effect of a change in the ambiguity of a skill, and study what our model predicts about how these changes influence positive self-image.

[10] The studies use intuitive notions of ambiguity. Felson (1981) compares an athlete's assessment of relative speed to football sense. Dunning et al. (1989) compare a subject's reported punctuality to his or her sensitivity.

In our formulation, ambiguity is measured by the extent to which different people have different interpretations of the same question. First consider the effect of changing the degree to which a particular skill is valued throughout the population. More precisely, assume that the components of λ are distributed independently of each other. Consider the effect of a mean-preserving spread on the i^{th} component of λ. Intuitively, this change leads to a wider range of opinions about the relative value of skill i. The first result gives a sense in which this kind of change increases positive self-image.

PROPOSITION 3: *In the CES model, if the components of λ are distributed independently of each other, a mean-preserving spread in the distribution of any component of λ increases $E_\lambda D^*(I, \lambda)$ for each I.*

Proposition 3 asserts that increasing the variability of the technologies increases an individual's subjective assessment of his ability relative to the population average.[11]

Suppose that originally everyone in the population had the same view of the marginal productivity of skill i. That is, λ_i was a constant. Proposition 3 implies that the expected ability gap, a measure of the amount of positive self-image in the population, increases when one introduces variation in the population about the relative importance of a skill i.

Another way to measure how ambiguity influences positive self-image is by comparing the change in positive self-image that arises when the number of skills increases. It is tricky to formulate this type of comparative-statics question, because adding a dimension to the set of skills changes the domain of technologies. We propose one variation to Proposition 3 for conducting this exercise that captures our intuition for why abilities that depend on many skills are associated with greater positive self-image.

Our intuition is that adding skills increases the number of ways in which individuals may differ in the subjective valuation of skills. Consequently, from one individual's perspective,there are more ways for others to make "incorrect" investments. We make this point for a special case of the CES model. Assume that each λ takes the form

$$\lambda_i = \begin{cases} 1 & \text{if } i \in V \\ 0 & \text{if } i \notin V \end{cases} \tag{16}$$

where V is a subset of cardinality v of the set of skills. That is, each individual believes that the skills in V (and only the skills in V) contribute to ability and that they contribute equally. Individuals differ in their income and in their opinion about which skills are valuable. Assume that all subsets of cardinality v are equally likely to be perceived as valuable. We now have a tractable framework in which to investigate the effect of increasing the total number of skills.

For this special case, individuals allocate their income equally over the skills in V. An individual with income I who believes that the skills in V are valuable will perceive himself to be better than an individual with income I' who believes that the skills in V' are valuable, provided that

$$I' \leq I\left(\frac{v}{w}\right)^{1/\rho} \tag{17}$$

where w is the cardinality of the intersection of V and V'. (When $w = 0$ the right-hand side of (17) is infinite.) When the total number of skills is n, the probability that two individuals have exactly w skills that they both perceive as important is

$$\frac{\binom{v}{w}\binom{n-v}{v-w}}{\binom{n}{v}}. \tag{18}$$

It follows from (17) that the smaller is w, the greater the (I, V)-individual's positive self-image relative to other individuals. It follows from (18) that decreasing n leads to an increase (in the sense of first-order stochastic dominance) of the distribution of w. That is, the larger the number of skills, the smaller the probability that another individual will share at least w favorite skills. This is precisely the

[11] Unlike positive self-image, the expected ability gap is not invariant with respect to ordinal transformations of technologies. Proposition 3 depends on our specification of the technology, in particular the property that the population's expected ability, $E_{(I',\lambda')}\{T(\phi(\lambda', I'); \lambda)\}$, is linear in λ.

result we wanted: increasing n leads to an increase in positive self-image.

PROPOSITION 4: *In the CES model, assume that* λ *satisfies* (16), *where V is equally likely to be any set of skills of size v. As the total number of skills increases, positive self-image of each individual increases.*

Proposition 4 depends on several special assumptions. Assuming that there is a set of valuable skills makes the comparison across different technologies easy to do. Assuming that only skills in V contribute to total ability keeps the amount invested in important skills constant (at I/v) as the number of skills increases. If we assumed that $\lambda_i > 0$ for all skills, then if income remained constant as the number of skills increased, individuals would reduce their investment, which could lead to a decrease in the level of positive self-image.

When λ satisfies (16), agents with different technologies invest in different subsets of skills. The consequence of different investment choices depends on the degree to which the skills are substitutes. For example, if one agent believes that ability depends only on the first two skills while another agent believes that ability depends on the second and third skill, then the first agent's perception of the second agent's ability will increase as skill 2 becomes a better substitute for skill 1. The parameter ρ is a measure of the extent to which one skill can substitute for another in the CES model. This reasoning suggests that positive self-image in the population increases as ρ decreases, an intuition confirmed by the following result.

PROPOSITION 5: *In the CES model, assume that* λ *satisfies* (16), *where V is equally likely to be any set of skills of size v. As* ρ *decreases, positive self-image of each individual increases.*

Proposition 5 follows immediately by differentiating (17). The result holds under more general conditions on the distribution of technologies, in particular if $\lambda_i = c$ for all $i \notin V$, $c \in (0, 1)$.

V. Control and Difficulty of Tasks

There is substantial evidence that positive self-image is more pronounced when tasks are easy than when tasks are hard.[12] For example, Justin Kruger (1999) asked students to make self-assessments of four abilities where the threshold for successful performance is low (using a mouse to operate a computer, driving, riding a bicycle, and saving money) and of four abilities where the threshold for successful performance is high (telling jokes, playing chess, juggling, and computer programming). Each student was asked to rank himself in a percentile for each ability. Students exhibited positive self-image with respect to the first set of abilities and negative self-image for the second set.[13] In experiments performed by Erik Hoelzl and Aldo Rustichini (2005), Don A. Moore (2002), and Moore and Tai Gyu Kim (2003), subjects chose between getting a payment based on pure chance (the roll of a die) or a payment based on their relative performance on a test. These studies found that, other things held constant, subjects were more likely to select a payment based on relative performance for easy tasks than for hard tasks, suggesting that there was greater positive self-image for the easier tasks.

This section investigates the idea that easier tasks tend to create more positive self-image. We take the position that easy tasks are those for which individuals have more opportunities to increase their ability. That is, we associate the difficulty of a task with the control an individual has in developing expertise.

Assume that a strictly increasing function $g : \mathbb{R} \to \mathbb{R}$ satisfying $g(0) \geq 0$ transforms income. If the cumulative distribution function of income is $F(\cdot)$ in the original population, then the transformed income distribution is $\tilde{F}(\cdot)$ where $\tilde{F}(\tilde{I}) = F(g^{-1}(\tilde{I}))$. Let $\tilde{B}(p)$ be the fraction of the transformed population that perceives itself to be in the top p-cile of the transformed population. From the definition of self-image, if $\tilde{B}(p) \geq B(p)$ for all $p \in [0, 1]$, then the transformation increases self-image.

[12] This observation is restricted to *relative* comparisons. Note that there is more opportunity to overestimate one's absolute ability when performance is low, which we expect in hard tasks.

[13] Kruger measured the degree of ambiguity of each ability and found that difficult abilities were considered more ambiguous than easy ones, which implies that the arguments of Section IV do not explain the experimental findings.

PROPOSITION 6: *In the CES model, if the income distribution is transformed by the function* $g : \mathbb{R} \rightarrow \mathbb{R}$, *where* $g(\cdot)$ *is strictly increasing, concave, and satisfies* $g(0) \geq 0$, *then the transformation increases the self-image of every individual.*

One function that satisfies the assumptions of Proposition 6 is a simple (rightward) shift in the distribution of income: $g(I) = I + c$, for $c > 0$. This transformation is equivalent to giving every member of the population extra income equal to c. It has the natural interpretation of giving individuals more opportunity to increase their skills. This finding is consistent with the results of several studies.[14]

Similarly, a shift to a higher (first-order stochastically dominating) distribution of income plays the role of making ability easier to obtain. An appropriate concave transformation of income generates an increase in the distribution of income. Suppose that $g(I) \geq I$ for $I = \underline{I}$ and $\bar{I}$. It follows that if $g(\cdot)$ is concave, then $g(I) \geq I$ for all $I \in [\underline{I}, \bar{I}]$. Consequently, since $\tilde{F}(\tilde{I}) = F(g^{-1}(\tilde{I}))$, if $\tilde{F}(\tilde{I}) = F(I)$, then $\tilde{I} = g(I)$ and hence $\tilde{I} \geq I$. It follows that $F(I) \geq \tilde{F}(I)$ so $\tilde{F}(\cdot)$ dominates $F(\cdot)$.

VI. Subjective Importance of Skills

Dunning et al. (1991) found that the extent to which individuals consider a skill to be important to describing a specific ability is positively related to the extent that they perceive themselves as having the skill. That is, individuals tend to claim that the skills that they possess are valuable.[15] For example, subjects first received a list of attributes associated with intelligence (vocabulary, logical reasoning) and were asked: "Which of the following are the best examples of intelligence?" A week later the same subjects were asked to rate how descriptive these attributes were of themselves. The correlations between productive attributes and perceived attributes were found to be positive.

The CES model predicts a relationship between skills and abilities consistent with the empirical findings. Two ways to formulate the result are completely straightforward. In the CES model, individuals invest in those skills that are relatively more productive. Consequently, the expected level of the more productive skills will be greater than the expected value of less productive skills. That is, if $\lambda_i > \lambda_j$, then $k_i^*(I, \lambda) > k_j^*(I, \lambda)$ and therefore (since λ is distributed independently of I) the inequality holds in expectation (over income) as well. This result does not require the assumption that technologies are CES. More generally, if skills are complements and increasing λ_i leads to an increase in the marginal productivity of skill i, then (under mild regularity conditions) increasing λ_i leads to an increase in k_i. Loosely speaking, making a skill more important leads to an increase in the final quantity of the skill.[16] Similarly, the expected level of the most productive skill (associated with the largest component of λ) is greater than the population's expected value of the skill.

There is a related result that is slightly less obvious. Just as it is possible to evaluate an individual's relative positive self-image in ability, it is possible to compute the individual's self-image with respect to a particular skill. For a given skill i, let

$$SI_i(I, \lambda) = \mu\{(I', \lambda') : k_i^*(I', \lambda') \leq k_i^*(I, \lambda)\}$$

be fraction of the population that has no more of the i^{th} skill than an (I, λ)-individual. $SI_i(I, \lambda)$ is the fraction of the population with a smaller amount of skill i. Note that $SI_i(\cdot)$ is not subjective: individuals are being compared along a single dimension. Subjectivity enters if individuals can select the skill over which they make comparisons. We wish to make precise the intuition that the ability to choose the skill over which comparisons are made increases positive self-image. One conjecture is that

$$\max_i SI_i(I, \lambda) \geq SI(I, \lambda). \tag{19}$$

[14] Mark D. Alicke (1985) and Dunning (1993) present evidence that positive self-image increases with the degree to which people claim to be able to control the trait.

[15] The empirical findings identify a correlation between possessing a skill and claiming that the skill is important. It is not clear whether individuals acquire the skill because they think it is important, or think the skill is important because they have it.

[16] The formal argument is an application of Aaron S. Edlin and Chris Shannon (1998, Theorem 5).

Inequality (19) states that an individual's self-image relative to his best skill exceeds his overall self-image. We are unable to prove this result in full generality, but can prove a variation.

Inequality (19) holds when an individual compares himself to individuals with a fixed alternative technology. To state this result, let

$$SI_i(I, \lambda|\lambda') = \mu(\{(I', \lambda') : k_i^*(I', \lambda') \leq k_i^*(I, \lambda)\}|\lambda')$$

be the relative self image in skill i conditional on λ'.

PROPOSITION 7: *In the CES model, for each* λ',

$$\max_i SI_i(I, \lambda|\lambda') \geq SI(I, \lambda|\lambda').$$

We know that in the skill-acquisition model, individuals have positive self-image. Proposition 7 states that, provided that individuals compare themselves only to members of the population with the fixed technology λ', every individual has a still greater positive image of himself with respect to a particular skill. This result is weaker than (19) because it permits an individual to use a different skill when comparing himself against different subpopulations.[17] When there are only two technologies in the population, Proposition 7 implies (19), because if $\lambda' \neq \lambda$, then $SI(I, \lambda|\lambda') \geq SI(I, \lambda)$.

It is also possible to establish (19) if there is a $c > 0$, such that every λ is of the form:

$$(20) \qquad \lambda_i = \begin{cases} 1 + c & \text{if } i = i^* \\ 1 & \text{if } i \neq i^* \end{cases}$$

for some i^*. If λ is always in this form, then each individual has one skill that he perceives as more valuable than all of the others.[18] To see that it is not possible to establish (19) without some assumption on the range of technologies, consider the case in which there is an individual with λ_i independent of i. If this individual has the median income, then one can find distributions on technologies where the individual's self-image is one-half for each skill. The individual will have a higher overall self-image, because he will perceive himself to have higher ability to others with higher incomes who do not invest equally in all skills.

VII. Objective Baselines

Many experiments demonstrate that people who perform the worst at tasks are the most likely to overestimate their ability. This finding is a statement about an individual's self-appraisal relative to an objective baseline. Although comparisons in our model are subjective, in the CES model, income is an obvious benchmark.

It is natural to measure one's self-image relative to the initial endowment. Specifically, define *relative self-image,* $R(I, \lambda) = SI(I, \lambda)/F(I)$. $R(I, \lambda)$ is ratio of the fraction of the population that an individual with technology λ and endowment I perceives to have lower ability than he does to the fraction that has lower income. The second term is an objective measure of this individual's standing in the population.

The revealed-preference argument used to establish Proposition 2 demonstrates that

$$(21) \qquad R(I, \lambda) \geq 1,$$

with a strict inequality if the technologies are distinct, so that all individuals perceive that their ability is greater than their relative income. We wish to investigate when $R(\cdot)$ is decreasing in I so that relative self image is declining in ability. Since $SI(I, \lambda) \leq 1$ and $F(\bar{I}) = 1$, it follows from (21) that $R(\bar{I}) = 1$. Since $F(\underline{I}) = 0$, it follows from (21) that the relative self-image of the lowest-ability individuals must exceed that of the highest-ability individuals. The ability to make investments permits all individuals to enhance their subjective relative standing. For those individuals with large initial income, however, there is not much room for improvement. These individuals have objective reasons for viewing themselves as better than most of the population. Agents with lower initial abilities stand to gain more from their investment. This simple argument demonstrates that "on average" relative self-image must decline with income, and seems consistent with the psychology literature.

[17] Similarly, the importance of the restriction to CES technologies is that, under these assumptions, the best skill is independent of income.

[18] The result follows from a straightforward calculation using (14), which we omit.

As income rises, an individual's objective ranking improves at a rate equal to the density of individuals with similar income. On the other hand, his perceived ability increases roughly in proportion to the number of individuals with higher incomes and different preferences. It follows that as income increases, relative self-image will tend to decrease if there are relatively fewer high-income individuals than low-income individuals. This suggests that global monotonicity of relative self-image would hold under strong assumptions on the shape of the distribution of income $F(\cdot)$. The next result formalizes the intuition. To state the result, let $F'(\cdot)$ be the derivative of $F(\cdot)$ and let $E(x) = (xF'(x))/F(x)$ be its elasticity.

PROPOSITION 8: *In the CES model, if $E(\cdot)$ is decreasing, then $R(I, \lambda)$ is decreasing in I.*

Proposition 8 states that relative to an objective ability standard (endowment), positive self-image is decreasing in ability provided that the elasticity of the distribution of income is decreasing. The assumption that $E(\cdot)$ is decreasing is strong, but holds (weakly) for the uniform and Pareto distributions.

An alternative modeling approach is to assume that there is an objective technology $T_0(\cdot)$. It is straightforward to obtain relative rankings by comparing $SI(I, \lambda)$ to the ranking an individual with characteristics (I, λ) would obtain using the technology T_0. We can show that individuals who rank near the top of the population with respect to the objective technology have lower positive self-image than individuals who rank near the bottom of the population. This approach may capture another experimental finding. Kruger and Dunning (1999) find evidence that the self-image of high-performing individuals is lower than their objective performance. This finding is consistent with the approach of using an objective benchmark. Agents who have high ability relative to the objective standard T_0 must have initial endowments of skills that produce high ability according to T_0. These skills may be less highly valued by the individual's own production function.

VIII. Further Implications and Tests

This section describes several implications of our model that could distinguish the model from alternative descriptions of behavior.

One alternative way to describe positive self-image posits that individuals have the ability to tailor their technology to their skills. In a model of self-enhancing evaluations, individuals differ in their skills and in their technology. Individuals adjust their technology to maximize their ability holding their endowment fixed. The arguments that we use to establish Proposition 2 work for this model, so that positive self-image will arise. Analogs of most of our results would exist in this model.

Another model assumes that individuals have an intrinsic desire to feel good about themselves, and they make systematic errors in information processing to support this desire. Psychologists often suggest this type of model.[19] We have not seen a formal description of this kind of model in the literature and do not offer one here. To the extent that this story relies on modifying the way one evaluates skills, it is similar to the model of self-serving evaluations. To the extent that this story relies on modifying beliefs (suppressing negative signals or overemphasizing positive signals), it is outside of our framework.

The alternative models provide predictions that are clearly different from our model's in four situations: fixed versus variable characteristics; heterogeneity in endowments; negative self-image; and experience. We discuss these situations now.

Some abilities can be changed, others are fixed. Our model predicts positive self-image with respect to abilities that can be increased through investment, but not with respect to fixed abilities. For example, a basketball player can practice taking free throws, passing, or dribbling, but has little control over his height. If you ask basketball players how important height is to the game, a theory of positive self-image based on self-serving evaluations predicts that the taller players would view height as a relatively more important factor than do smaller players. Motivational models that predict positive self-image because individuals are more likely to pay attention to positive information will not distinguish between fixed and variable characteristics. We are not familiar with any study that tests the sensitivity of positive self-image to the degree in which skills can be varied. The experiments that come closest to asking this question demonstrate that positive

[19] Dunning (1993) is one example.

self-image increases with the level of control that an individual has. Provided that the quantity being controlled is investment in skill enhancement (rather than ability to adjust technology), these results support our model.

Proposition 3 suggests that positive self-image will be stronger when subjects are asked to compare themselves with groups that are more diverse in the sense that they have access to a wider range of technologies. Surveys that carefully vary the comparison group should (according to our model) generate systematic changes in the degree of positive self-image, with positive self-image decreasing the more homogeneous the technologies of the comparison group. One would expect the same result for a model with self-serving evaluations.

We can also measure heterogeneity through differences in the endowments. Here, the alternate models provide different predictions. If individuals select technologies to enhance their assessment of their initial skills, then there would be little positive self-image in a population with similar initial skills. In the limiting case where all endowments are identical, choice of technology will not permit anyone to perceive his ability to be better than anyone else's. This prediction is not consistent with the widespread observation of positive self-image. In contrast, in our model, when there is no variation in the initial endowment, the strongest possible kind of positive self-image effect exists: everyone perceives himself to be the best. In general, there is a sense in which greater variability in income decreases positive self-image in the skill acquisition model. The idea is that skill augmentation allows certain individuals to become better than others by making an appropriate investment. The investment has only limited ability to improve one's subjective ranking. It only permits an agent to get better than individuals who have similar endowments. By making endowments more variable, the chance of moving up in relative rankings through investment decreases. The next proposition states the result precisely.

PROPOSITION 9: *In the CES model, a mean-preserving spread in the distribution of income decreases the expectation of the ability gap with respect to income for each* λ.

Proposition 9 follows because the expected ability gap is an increasing concave function of income. We are not familiar with any studies that test the prediction of Proposition 9.

While we have concentrated on situations that exhibit positive self-image, our model suggests situations in which we would expect to see negative self-image. We assume that every individual in the population has the same ability to augment her skills. Our model predicts that individuals who are relatively less able to improve their skills have lower self-assessments. There is evidence (Lauren B. Alloy and Lyn Y. Abramson, 1979) that individuals who claim to have less control have lower self-images. The self-serving assessment approach would generate the same conclusion, provided that control were interpreted as the ability to change one's opinion about the true technology.

In the skill-acquisition model, we assume that initial skills are distributed independently of technologies. On the other hand, Proposition 1 demonstrates that, in the general model, if skills were distributed so that people had relatively low endowments of the skills that they thought were most productive, then negative self-image would result. We see no reason why endowments should be negatively correlated with productivity, so we do not think that this is a compelling reason for negative self-images. It does suggest that one could induce negative self-image in the laboratory by selecting individuals whose endowments do not match their productivity, by asking individuals to rate themselves on the basis of attributes selected by others,[20] or by asking individuals to base ability rankings on dimensions that they earlier described as unimportant. If individuals evaluate ability using a technology tailored to their skills, then they would continue to exhibit positive self-image in these circumstances. Motivational models of egocentric bias also predict that subjects overestimate their ranking under these conditions, so our model is the only one that allows for the possibility of negative self-image.[21]

Our model predicts that self-image increases

[20] Dunning et al. (1989) show that self-image goes down when people evaluate themselves using lists of skills created by others rather than themselves.

[21] Kruger (1999) finds negative self-image for difficult tasks and attributes this in part to subjects having better information about their own ability than the ability of others.

with experience, provided that investment opportunities increase with experience. This prediction would be in contrast to rational learning models where experience with outcomes could lead to convergence of individuals' beliefs to objective beliefs about ability. There is some evidence from studies about perceptions of driving ability that positive self-image is increasing with drivers' experience.[22] Positive self-image would not necessarily increase with experience in a model of self-serving assessments.

IX. Related Literature

Some examples of positive self-image are easily seen to be consistent with unbiased information processing and rational decision making. For example, if drivers are either good or bad, and the only informative signal about driving ability is whether a driver is involved in a serious accident, then (provided serious accidents are rare) most drivers will never be involved in an accident and correctly view their ability as above average. Isabelle Brocas and Juan D. Carrillo (2004) and Ján Zábojník (2004) present dynamic models in which this bias arises when utility-maximizing individuals have an opportunity to learn about their abilities through costly experimentation. Experimentation has the potential advantage of providing information that individuals can use to make a better decision; it has the cost of postponing the time of the decision, and (due to discounting) reducing the value of the payoff. Brocas and Carrillo observe that information about an action whose payoffs have a high variance is more valuable; consequently, an agent is more likely to experiment when her prior favors the less risky decision. In Zábojník's model, the payoff is an increasing function of ability. Consequently, the opportunity cost of experimentation is higher for agents who have higher priors on their ability. Zábojník shows that only individuals with low self-assessments continue experimenting, which creates a bias in the distribution of posterior beliefs. An implication of the bias is that (under appropriate conditions) the fraction of agents who believe their ability to exceed a cutoff value is larger than the objective fraction. These papers present plausible, rational mechanisms through which a population's beliefs may become skewed over time. In the models of Brocas and Carrillo, Zábojník, or in the simple model of drivers, the fraction of people who place themselves in the top p percent of the population will not exceed p for all p. That is, the positive self-image effect is weaker than the one we identify in Proposition 2.

There is a small number of studies that investigate positive self-image or related biases using economic incentives. Hoelzl and Rustichini (2005), Moore (2002), and Moore and Kim (2003) examine positive self-image in an experimental setting with monetary incentives. While their designs differ, the papers all identify a subject's beliefs about relative standing by asking the subject whether a reward should be based on a test of skill or the outcome of a random device. The experiments reveal positive self-image when more than half of the subjects prefer to be rewarded on the basis of their performance on the test of skill than on the basis of a randomization device that selects a winner with probability one-half. These papers observe that the extent of positive self-image increases when the test becomes easier (and even find evidence of negative self-image when relative performance on difficult tests determines monetary payoffs). More people voted for performance-based payment for easier tasks than for hard ones. Monetary payments significantly reduced positive self-image. Our model applies to these studies only to the extent that different subjects can hold different beliefs about how individual characteristics translate into performance on the test of skill. If individuals agree on the "production function" that determines test performance, have unbiased information about the inputs to this production function, and seek to maximize monetary payoffs, then our model implies that there will be no positive self-image.

Colin Camerer and Dan Lovallo (1999) study an entry game. Payoffs in the game are based on rank, which is determined either randomly or through a test of skill. There is more entry when relative skill determines payoffs, which suggests that individuals overestimated their ability to do well on the test relative to others. As with

[22] Svenson et al. (1985) describe a study of Spolander that obtained self reports of driving skill from individuals with experience of one month, one year, and three years. The least experienced drivers exhibited a below-average effect, positive self-image appeared in the group with one year of experience, and was still greater in the most experienced group.

the papers of Moore (2002) and Moore and Kim (2003), our model can explain these findings only to the extent that different subjects disagree about what skills contribute to success on the test and that subjects have made investments appropriate to the test. Since subjects made entry decisions prior to taking the test, it is plausible to assume that different subjects had different notions of what ability was being tested. It is harder to see why subjects would believe that the skills that they had chosen to improve were relevant to the experimental task. Camerer and Lovallo argue that subjects fail to take into account the strategic behavior of their opponents—in particular, subjects neglect that self selection leads only the most highly skilled subjects to enter when payments are based on skill.

X. Conclusion

We demonstrate that, in the presence of skill enhancement, egocentric comparisons lead to positive self-image. We do not explain why individuals make egocentric comparisons.[23] Our model organizes a range of observations under the assumption that people make egocentric comparisons.

We are in agreement with Dunning (1993, p. 99) when he writes, "The central tenet guiding the discussion is that people are often not referring to the same actions and characteristics when invoking the same word or concept," and are convinced by the finding that optimism and positive self-image are widespread. This paper provides a descriptive model and suggests that positive self-image may not be a compelling reason to change modeling approaches.

While our conventional model captures much of what has been classified as biased behavior, we have discussed some observations that are not consistent with our model. Moreover, two biases that are difficult to separate from positive self-image are widespread but not explained by our model. It would be valuable to pursue the extent and implications of the finding that (nondepressed) individuals have an exaggerated estimate of their ability to control events and also the inability to take into account strategic behavior of others (as identified in the papers discussed in Section IX).

All of our results require strong assumptions. By investigating environments where the conditions in our propositions fail, we are led to situations where the commonly observed qualitative properties of relative self-image are unlikely to hold.

[23] Psychologists present two reasons for egocentric comparisons. Motivational arguments posit that individuals get utility directly from maintaining positive feelings about themselves. Our work explains why egocentric comparisons supply positive feelings. Cognitive theories assume that it is easier to categorize behavior according to something familiar (personal preferences) than something less familiar (the preferences of another), and so egocentric comparisons conserve scarce information-processing costs.

Appendix

PROOF OF PROPOSITION 1:

Fix λ. For $p \in [0, 1]$, let u_p satisfy

$$u_p = \inf\{u : \mu(\{k : u \geq T(k, \lambda)\}|\lambda) \geq 1 - p\}. \tag{A1}$$

Let k_p satisfy $T(k_p, \lambda) = u_p$. k_p exists for all p by continuity of $T(\cdot)$ and compactness of C. From (A1),

$$SI(k_p, \lambda|\lambda) = \mu(\{k : T(k_p, \lambda) \geq T(k, \lambda)\}|\lambda) \geq 1 - p. \tag{A2}$$

Let $B(p|\lambda') = \{k : SI(k, \lambda|\lambda') \geq 1 - p\}$. It follows that

$$\text{if } SI(k, \lambda|\lambda') \geq S(k, \lambda|\lambda), \text{ then } B(p|\lambda') \supseteq B(p|\lambda). \tag{A3}$$

Since $T(k, \lambda) \geq T(k_p, \lambda)$ implies that $SI(k, \lambda|\lambda') \geq SI(k_p, \lambda|\lambda')$, $B(p|\lambda')$ contains $\{k\text{: } T(k, \lambda) \geq T(k_p, \lambda)|\lambda')\}$ and therefore (A1) implies that

$$\mu(B(p|\lambda)) \geq p. \tag{A4}$$

Since $B(p) = E_{\lambda'}\{B(p|\lambda')\}$, it follows from (A3) and (A4) that $SI(k, \lambda|\lambda') \geq S(k, \lambda|\lambda)$ implies that

$\mu(B(p)) \geq p$. Further, if there exists λ and λ' for which the inequality in (10) is strict for almost all k, then positive self-image must be strict. A symmetric argument establishes the result for negative self-image.

PROOF OF PROPOSITION 2:

By Proposition 1, it suffices to show that

$$\text{(A5)} \qquad SI(k, \lambda|\lambda') \geq S(k, \lambda|\lambda).$$

Let $W(k, \lambda|\lambda') = \{I' : T(\phi(I, \lambda'), \lambda) \geq T(k, \lambda)\}$. By definition,

$$\text{(A6)} \qquad SI(k, \lambda|\lambda') = F(W(k, \lambda|\lambda')).$$

Since $T(\phi(I, \lambda), \lambda) = T^*(I, \lambda) \geq T(\phi(I, \lambda'), \lambda)$ (with the inequality strict if $\phi(I, \lambda) \neq \phi(I, \lambda')$), it follows that for all k

$$\text{(A7)} \qquad W(k, \lambda|\lambda) \subset W(k, \lambda|\lambda')$$

with strict containment if $\phi(I, \lambda) \neq \phi(I, \lambda')$ for some I satisfying $T^*(I, \lambda) = T(k, \lambda)$. Inequality (A5) follows from (A6) and (A7), implying the result.

PROOF OF PROPOSITION 3:

The proposition asserts that a mean-preserving spread in the distribution of the i^{th} component of λ increases $E_\lambda\{D^*(I, \lambda)\}$ for all I. Since mean-preserving spreads increase the expectations of convex functions, it suffices to show that $D^*(I, \lambda)$ is convex in λ_i for all i. Direct computation shows that $T^*(I, \lambda)$ is convex and that $ET(\phi(I', \lambda'); \lambda)$ is linear in λ_i. Since $D^*(I, \lambda) = T^*(I, \lambda) - ET(\phi(I', \lambda'); \lambda)$, the proposition follows.

PROOF OF PROPOSITION 6:

To prove the result, it is sufficient to establish $\tilde{B}(p) \geq B(p)$ for all $p \in [0, 1]$, which holds, provided that $\widetilde{SI}(g(I), \lambda) \geq SI(I, \lambda)$ for all I and λ, where $\widetilde{SI}(\tilde{I}, \lambda)$ is the fraction of the transformed population that a $(\tilde{I}, \lambda)$-individual perceives as less skilled in the trabnsformed population.

Positive self-image will be greater in the new population if

$$\text{(A8)} \qquad \tilde{F}(g(I)h(\lambda', \lambda)) \geq F(Ih(\lambda', \lambda)).$$

Since $\tilde{F}(g(I)h(\lambda', \lambda)) = F(g^{-1}(g(I)h(\lambda', \lambda)))$, and $g(\cdot)$ and $F(\cdot)$ are increasing, (A8) is equivalent to

$$\text{(A9)} \qquad g(I)h(\lambda', \lambda) \geq g(Ih(\lambda', \lambda)).$$

Since $h(\cdot) > 1$, inequality (A9) holds provided that $g(\cdot)$ is concave.

PROOF OF PROPOSITION 7:

We need to show that

$$\max_i SI_i(\lambda, I|\lambda') \geq SI(\lambda, I|\lambda').$$

Using the formula for $k_i^*(\cdot)$ in the CES model, we have

$$SI_i(\lambda, I|\lambda') = F\left(I\left(\frac{\lambda_i}{\lambda_i'}\right)^\sigma \left(\frac{\sum_{j=1}^{n} (\lambda_j')^\sigma}{\sum_{j=1}^{n} \lambda_j^\sigma}\right)\right).$$

Consequently, it is sufficient to prove that

$$\max_i \left(\frac{\lambda_i}{\lambda_i'}\right)^\sigma \geq h(\lambda, \lambda')\left(\frac{\sum_{j=1}^{n} \lambda_j^\sigma}{\sum_{j=1}^{n} (\lambda_j')^\sigma}\right).$$

Substituting the formula for $h(\cdot)$ this becomes:

$$\text{(A10)} \qquad \sum_{j=1}^{n} \max_i \left(\frac{\lambda_i}{\lambda_i'}\right)^{\rho\sigma} \lambda_j(\lambda_j')^{\rho\sigma} \geq \sum_{j=1}^{n} \lambda_j^\sigma.$$

Inequality (A10) follows since

$$\sum_{j=1}^{n} \max_i \left(\frac{\lambda_i}{\lambda_i'}\right)^{\rho\sigma} \lambda_j(\lambda_j')^{\rho\sigma} \geq \sum_{j=1}^{n} \lambda_j^{\rho\sigma+1} = \sum_{j=1}^{n} \lambda_j^\sigma.$$

PROOF OF PROPOSITION 8:

In the CES model, the relative standing of an individual with income I and technology λ is

$$E_{\lambda'}\left\{\frac{F(Ih(\lambda'; \lambda))}{F(I)}\right\}. \tag{A11}$$

In order to prove the proposition, it is sufficient to show that (A11) is decreasing in I for all λ and λ'. Differentiating (A11) with respect to I yields

$$\frac{hF'(Ih)F(I) - F(Ih)F'(I)}{F^2(I)}, \tag{A12}$$

which is nonpositive provided that $(xF'(x))/F(x)$ is decreasing.

PROOF OF PROPOSITION 9:

It suffices to prove that a mean-preserving spread in the distribution of income decreases $D^*(I, \lambda)$ for each λ. Direct computation shows that there exist functions $c(\lambda)$ and $d(\lambda)$ with $c(\lambda) > 0$ such that $D^*(I, \lambda) = c(\lambda)I^{\rho} + d(\lambda)$. Therefore, $D^*(I, \lambda)$ is concave in I and so a mean-preserving spread in the distribution of income decreases $E_I D^*(I, \lambda)$.

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