On the Relation Between the Endogenous Growth Rate of the Economy and the Dynamics of Renewable Resources *

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02.10.2007

Abstract

In this paper we study a simple endogenous growth model in which the two engines of growth are the exogenous technical progress in dematerialization and the accumulation of a renewable natural resource. The model is also labeled as been "*endogenous*" as the rate of growth of natural capital is endogenously determined and should lie between zero and the rate of technical progress. In this context, it is possible to combine permanent economic growth *with* permanent growth of the environmental asset.

the endogenous rate of growth of the stock of natural resources is a positive function of the physical rate of regeneration (which will occur if consumption would be zero) and of the rate of technical progress. However, in order to assure sustainability, the former growth rate should be larger than zero but smaller than the later. Second, the output growth rate (which in our model is equal to the rate of consumption) should lie between the rate of technical progress and the sum of the rate of technical progress and the natural rate of regeneration. Therefore, even in the case in which the physical rate of renewal is mall, this will allow for unbounded growth. Third, in our simple model, there is no transitional dynamics.

Keywords: endogenous growth, environmental preservation, habit-formation.
JEL codes: C61, Q56, O39, O40.

*The research presented in this paper has been financially supported by the *Fundação para a Ciência e a Tecnologia*, under grant POCTI/eco/13028/98.

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1 Introduction

One of the most remarkable features of human kind evolution has been its ability to generate permanent growth. Not surprisingly, due to the production's dependency on energy and/or material, this stylized and striking fact has also been followed by the increasing use of natural resources. Given the earth's material finitude (although it is an open system from the energy point of view) and the link and feedbacks between economic and natural systems, the obvious question is whether it is possible to combine (permanent) economic growth and environmental preservation.

The "mainstream" literature of the 70's and the 80's, mainly based on the prevailing neoclassical growth framework, was essentially focused on the limits imposed by the scarcity of natural resources which, in turn, were basically seen as inputs of production (Dasgupta and Heal 1979; Stiglitz 1974). The results showed that sustained growth might be feasible even under conditions where natural resources are exhaustible, in limited supply, essential for production and with positive population growth (Stiglitz 1974).

By that time some authors (e.g. Boulding (1966), Kneese and Arge (1970), Daly (1973), Georgescu-Roegen (1971), Georgescu-Roegen (1975), Hardin (1968)) began to highlight the economic relevance of the thermodynamic laws and namely on the potential limits that physical and natural processes impose on economic activity and on the difficulties in invoking the price mechanism, especially in view of the public nature of natural assets. These contributions were responsible for a re-orientation of some environmental thinking during the eighties, namely the recognition that knowledge accumulated by natural sciences could be used and applied to both economic processes[1] and economic thinking. Although within the neoclassical paradigm, a growing body of research was then devoted to the study of the impacts of including the accumulation of pollution and its disutility. The main results suggested that with exogenous technical change pollution would accumulate in the environment as long the economy grows, and the productivity of physical capital approaches zero.

The incorporation of environmental considerations into economic growth thinking received a new incentive when, by the end of the eighties, a new class of economic growth models (known as "endogenous growth models") emerged after the work of Lucas (1988), Romer (1990), Grossman and Helpman (1991) and Rebelo (1991)[2] and for whom techno-

[1] For example, the rate and scale of throughput passing through the economic system is subject to an entropy constrain.

[2] Which, in turn, were inspired by Backer´s (Becker (1964)), Uzawa's (Uzawa (1965)) and Nelson and Phelps's (Nelson and Phelps (1966)) theories of human capital

logical change, knowledge and human capital are seen as internally dynamic, endogenous sources of economic growth.

In the last two decades many papers have been written on the relationship between economic growth and environmental preservation within the endogenous growth framework[3]. Either predicting "ecologically unsustainable growth"[4] (see, for example, Michel and Rotillon (1995), Mohtadi (1996), Stokey (1998)) or "ecologically sustainable growth" (see, for example, Musu (1994), Musu (1995), Gradus and Smulders (1993), Smulders and Gradus (1996), Bovenberg and Smulders (1995), Xepapadeas (1997), Belbute (1999), Barbier (1999), Chevé (2000) and Rubio and Aznar (2001), most of those studies share the common assumption that natural assets are limited and subject to diminishing returns as a result of biophysical laws (especially the thermodynamic laws) that governs them. Therefore, economic growth based on resource use can only the sustained unless technological progress is unbounded and if natural inputs and man-made capital are good enough substitutes. Substitution and technological progress are, indeed, the "...economic forces that shape the interaction between growth and scarcity" (Smulders (2000)). So, the basic idea underlying these approaches is that permanent economic growth is feasible because it is fueled by the growth of man-made capital.

This paper explores a different perspective. Although recognizing that the accumulation of human knowledge represents a key factor for the continuous expansion of the economy within the limited physical system of the earth (Smulders (1999)), the paper explicitly assumes that natural resources might be for themselves an additional endogenous source for growth. This possibility has not been addressed in the literature of sustainable endogenous growth but has been implicitly suggested by Smulders (2000) by recognizing that "*...the long-run growth rate ... depends on technology parameters, preference parameters, and environmental parameters (the parameters of the regeneration function)*".

In order to illustrate the basic idea let us use the case of energy as example. Energy plays a crucial role throughout the world for both consumption and production activities: lighting, heating, cooling, cooking, transportation, and in virtually all productive activities. In fact, without it life will eventually cease. Energy can be generated from a variety of sources (coal, crude oil, natural gas, nuclear power, water, wind, solar light etc), some of which are depletable and nonrecyclable while others are not.

Technological progress can help to overcome the physical limits of some of these

[3] See Smulders (1999) or Xapapadeas (2003) for a review of the literature in endogenous growth and the environment

[4] Essentially because growth is accomplished with deterioration of environmental quality

energy sources but, until now it has been unable to generate a feasible substitute for energy. Clearly energy has no substitute and given the finite nature of some of its basic sources, technological changes will only be able to postpone the moment where these nonrenewable energy sources will be completely exhausted. Ultimately humankind energy needs will have to be fulfilled from a continuous supply of renewable energy sources.

On the other hand technological progress and innovations play an important role in improving the efficiency of the technologies which, in the long run, will lead to lower energy use intensity (dematerialization). However, the empirical reality of technological change shows that without regulatory intervention or adequate price signals/incentives, it is not clear that technological progress would be energy saving. Furthermore, an increase in energy efficiency necessarily implies a reduction in the unit cost of producing output, which leads to an increase in output, thereby increasing energy use. This "rebound effect", as it is known in the literature, can be quite significant (se, for example, Brannlund and Nordstrom (2007)). Technical progress cannot free humankind from the dependency of energy. We believe that the same applies to environmental resources. Even the most optimistic view about the role played by the "backstop technologies" in freeing Mankind dependency of natural resources, depends crucially on the availability of a continuous flow of renewable resources, even after the total depletion of nonrenewable resources. As humankind history has already shown, the need of natural resources has never ceased to grow until now.

This paper establishes the explicit link between the endogenous growth rate of the economy and the growth rate of the natural resource. We adopt a very simple structure of the model in order to focus our attention to the basic mechanism. The economy we have in mind displays endogenous growth patterns and exhibits two sources of unbounded growth (in consumption and utility): the growth of the renewable resource and the technical progress in dematerialization. We adopt a broad definition of natural capital in order to include renewable, exhaustible and environmental resources (Pearce and Turner (90 a)). This stock of natural capital has the ability to renew itself at a constant and positive rate. Apart from its productive properties, natural capital has also a direct and positive impact on consumer's well-being. There are no externalities and other distortion which implies that we may see this economy both as a decentralized or centralized.

Our conclusions are the following: First, the endogenous rate of growth of the stock of natural resources is a positive function of the physical rate of regeneration (which will occur if consumption would be zero) and of the rate of technical progress. However, in order to assure sustainability, the former growth rate should be larger than zero but smaller than the later. Second, the output growth rate (which in our model is equal to the rate of consumption) should lie between the rate of technical progress and the sum of the rate of technical progress and the natural rate of regeneration. Therefore, even in the case in which the physical rate of renewal is small, this will allow for unbounded growth. Third, in our simple model, there is no transitional dynamics.

The paper is organized as follows: Section 2 present a brief review of the literature on the extension of endogenous growth theory to the environmental preservation concerns. Sections 3 and 4 present the balanced growth path and the dynamics. Sections 5 and 6 show the effects of the technical and preferences parameters over the long run growth rate and section 7 concludes the paper.

2 The Model

Consider an economy that produces a single good and uses the stock of (man-made) knowledge A(t) as an input of production (Mankiw (2000)). Additionally the level of output is (negatively) affected by pollution, P(t) (see for example Rubio and Aznar (2001), Musu (1995), Tahvonen and Kuuluvainen (1991a), Smulders (1995)), which, in turn, is a result of a joint production. The negative impact of pollution on production because high levels of pollution render the economy to be less productive either because workers became less productive as a result of the effects of pollution in their health or because pollution reduces the biodiversity which reduces the potential of the production of new knowledge (Smulders (1999)).

$$Y(t) = A(t)P(t)$$

The index of technology (as A(t) is also often known) evolve exogenously accordingly to the following rule

$$A(t) = A(0)e^{\gamma_A t}$$

which captures the Jones (1995) (or even Jones (1998))'s argument that "*... ideas improve the technology of production*". Ideas are nonrivalrous in the sense that the use of one idea by one person does not preclude its use by another. Once an idea is created, it can be used by everyone at the same time, over and over. Moreover, any idea can be used by others to produce subsequent generations of ideas. Clearly, ideas create ideas . This characteristic of ideas implies the presence of increasing returns to scale. Nevertheless, ideas vary substantially in their degree of excludability. They are said to be nonexcludable when they tend to be freely available to everyone and thereby generating a large quantity of spillover (externalities) benefits that are unable to be fully captured by their producers. However, even when ideas (and the knowledge behind it) are mainly a public good, much of the search for a new idea is done in firms that are mainly profit-driven. This search is profitable since new ideas give firms temporary benefits (monopoly rent), either because they are the firsts on the market with a new product or because of the patent system.

Although the motivation for the production of new ideas depends on the degree to which firms are able to capture the benefits of their efforts to produce them, which in turn is the base of endogenous incentive for endless growth, we will use the simplest structure of production in order to focus our attention on the basic mechanisms behind the central idea of paper. In terms of the model we are using, we will follow Jones (1998) by assuming that any new idea is responsible for an increase in the technology index A(t) where γ_a a is the rate of growth of new ideas (or, equivalently, the rate of technological progress) which is assumed to be exogenous.

Natural processes are modeled as renewable resource which accumulate as a result of two counteracting processes; the natural regeneration which takes place at a constant rate μ (also known as the maximal potential rate of regeneration of the environment (see Tahvonen and Kuuluvainen (1991b)) and the depletion of resources as a result of the productive activity. Both the extraction of resources and the disposal of waste are captured by $P(t)$ because both actions reduces the stock of available environmental resources.

$$\dot{N}(t) = \mu N(t) - P(t), \quad N(0) = N_0, \tag{1}$$

There is an ongoing debate about the appropriate specific form of the natural regenerative capacity that has its origin on the economic relevance of the laws of thermodynamics, especially the law of entropy. The theme was first introduced by Nicholas Georgescu-Roegen (see for, example, Georgescu-Roegen (1971), Georgescu-Roegen (1977)) and by the main modern exponent of this thesis Daly (1973) and Daly (1992). According to this law (also known as the law of conservation of material and/or energy) no material or energy can be created in any closed system: only transformation takes place. Moreover, all available material or energy is transformed ultimately into useless heat due to entropic processes. However, since the hearth is not a closed system, the environment does not have to rely solely on its own services. Environmental resources can then be preserved thanks to the regular inflow of energy from the sun, which offsets the entropy process and allows for a steady "production" of ecological services. The supply of these “ecological” services is captured by a hump-shaped curve which represents the net amount of energy available for rival use. Given the fixed inflow of solar energy, diminishing returns apply.

Although some authors welcome this argument by assuming that natural regenerative processes are subject to diminishing returns as a result of the entropy law (Smulders (1995), Smulders (2000), Tahvonen and S.Salo (1996), Tahvonen and Withagen (1996), Chevé (2000), Belbute (1999), Belbute et al. (2005)), there are several reasons to use a linear representation of the regeneration process instead (seeMusu (1995), Le Kama and Schubert (2003), Tahvonen and Kuuluvainen (1993), Smulders and Gradus (1996), Li and Lofgren (2000)). The first is based on the so-called "net-energy" school (see, for example Weinberg (1977) and Weinberg (1978)) for whom the only scarce element that threatens the possibility of endless growth is the availability of useful energy (exergy), not the thermodynamic laws by themselves. Provided that both a sufficient energy flux from outside the system and a reservoir (or, to use Georgescu-Roegen own words, a "fund") of materials/wastes are available, the "spaceship economy" model (see Boulding (1966)) implicit in the previous view might be consistent "... with the second law of thermodynamics" (Ayres (1999), pp 480, see also Ayres (1997)).

Secondly, there is what can be called the Environmental Kuznets Curve Hypothesis (EKCH) argument. The EKCH states that environmental degradation and income should have an inverse-U relation (Grossman and Krueger (1995)). The factors that might explain the negative relation between those variables above some threshold level are scale, composition and technological change. As the economy grows, pollution and the demand for resources also grow (the scale effect), but if economic sectors with lower than average environmental impact grow above average (composition effect) and new cleaner technologies are invented (innovations), the overall environmental impact may decrease over time and the assimilative and regenerative capacity will tend to increase continuously (Belbute et al. (2005)).

The EKCH argument can be complemented by what could be called the "dematerialization argument"; due to both innovation (new technologies may be resource saving) and composition changes (less materialized sectors of society may grow faster than average), the material throughput per unit of income that crosses the economy, tend to decrease along time, and thus allowing natural assets to increase its regenerative and assimilative potential

Finally, a similar argument is used by Rosendahl (1996)) but for practical and numerical simplicity. He argues that the constant and positive rate of regeneration might be a practical approximation when simply the positively sloped arm of the hump-shaped regeneration function is relevant, at least for some moments in time (i.e. when the environment is far away from its "virgin state"). As an example he refers the case of developing countries in which the environment is already severely deteriorated and thus, it is reasonable to assume that the state of the environment may be way below some threshold level.

The equilibrium condition for the goods markets is

$$Y(t) = C(t) \tag{2}$$

as, for simplicity, we assume that there is no investment in physical capital (as in Rosendhal, 1996). Although this might be considered a controversial assumption for developed countries or regions, it has been argued that for most developing countries or regions not only the supply of physical capital is scarce but also the financial and institutional conditions make it difficult for people to get the scarce capital. We also do not consider the existence of abatement activities.We assume implicitly that if there is a environmentalist policy, it is directly performed by firms by controlling $P(.)$. We assume that the representative agent has the intertemporally independent utility function

$$V(\{C(t)\}_{t=0}^{\infty}, \{N(t)\}_{t=0}^{\infty}) = \int_0^{\infty} u(C(t), N(t))e^{-\delta t}dt$$

where $\delta > 0$ is the psychological discount rate and the instantaneous utility function, $u(.)$, is increasing and concave in both its arguments. The consumer derives utility not only from the services stemming from the consumption of the manufactured good but also from the amenity services produced by nature. A concave utility function means that there is some degree of substitutability (see Le Kama and Schubert (2003)).

There are several necessary conditions for the existence of a balanced growth path (BGP). First, the levels of consumption and natural resources should be written as

$$C(t) = c(t)e^{\gamma_c t},$$

and

$$N(t) = n(t)e^{\gamma_n t},$$

where c and n are the detrended variables and γ_c and γ_n are the long run growth rates. Second, from equation (1), we see that the growth rates of the stock of natural resources and of pollution should be the equal. Therefore $P(t) = p(t)e^{\gamma_n t}$. Third, the equilibrium condition in the goods market should hold. Then,

$$\gamma_c = \gamma_n + \gamma_a = \gamma$$

where γ is the growth rate of the output, and $c(t) = A(0)p(t)$.

Then, the detrended resource accumulation equation, becomes

$$\dot{n}(t) = (\mu - \gamma_n)n(t) - \alpha c(t) \tag{3}$$

where $\alpha \equiv A(0)^{-1}$.

The fourth condition for a BGP, is that the utility function should be homothetic (as is well know from Rebelo (1991) or Palivos and Wang (1996)). As we have a state variable in the utility function and the rate of growth of the two variables is not equal when there is technical progress, we assume that the utility function may be written in the form

$$u(C(t), N(t)) = e^{\gamma_u t} u(c(t), n(t)), \tag{4}$$

where

$$u(c, n) = \frac{(cn^{\varphi})^{1-\sigma}}{1-\sigma}$$

is the specific form for the instantaneous utility and $\varphi = \frac{u_n}{u_c}\frac{n}{c}$ measures the relative utility from the amenity services produced by natural capital as regards the services from the consumption of material goods. It can also be seen as the importance ascribed to the services provide by natural assets to wellbeing. σ is the coefficient of relative risk aversion of the standard CRRA and also the inverse of the instantaneous intertemporal elasticity of substitution. Moreover, γ_u is the growth rate of the utility index and it is described by

$$\gamma_u = (1-\sigma)(\gamma_c + \varphi\gamma_n) = (1-\sigma)(\gamma_a + (1+\varphi)\gamma_n).$$

Therefore, the intertemporal optimization problem for the centralized version of this economy is

$$\max_{\{c(t)\}_{t=0}^{\infty}} \int_0^{\infty} \frac{(c(t)n(t)^{\varphi})^{1-\sigma}}{1-\sigma} e^{-\delta^* t} dt \tag{5}$$

where

$$\delta^* = \delta - \gamma_u$$

subject to equation (3) and given $n(0) = n_0$.

The intuition behind this problem is the following. The growth rate of consumption depends on the growth rate of natural resources and the growth rate of technical progress. As in Belbute et al. (2005), we assume that new technologies generates a process of dematerialization, i.e., the possibility of producing more with a decreasing use of raw materials. In this first approximation, we assume that technical progress is exogenous, but it could be endogenized.

Some natural resources should been used in order to consume manufactured goods, but this decreases the amenity services produced by nature. Therefore, the optimal rate of growth of natural resources should belong to the interval $(0, \mu$ and is determined by the trade-off between consumption of manufactured goods and consumption of amenities. Note that the growth rate of the natural resource is different from the rate of natural renewal. The difference is $\gamma_n - \mu$.

Assumption 1 *Let* $0 < \sigma < 1$, $\varphi > 0$, $\mu > 0$ *and*

$$0 < \gamma_a < \frac{1}{(1-\sigma)} \left[\delta - (1-\sigma)(1+\varphi)\mu\right]$$

Assumption 2 *Let* $\varphi > 0$, $\mu > 0$ *and*

a) when $0 < \sigma < 1$,*then*

$$0 < \gamma_a > \delta - (1+\varphi)\mu \tag{6}$$

b) when $\sigma > 1$, *then*

$$0 < \gamma_a < \delta - (1+\varphi)\mu \tag{7}$$

Assumption 3 *Let* $\varphi > 0$, $\mu > 0$, *then*

$$0 < \gamma_a > \frac{\delta - (1+\varphi)\mu}{(1-\sigma)} \tag{8}$$

All the three previous assumptions are dependent whether the substitution between intertemporal consumption is inelastic ($\sigma > 1$) or elastic ($\sigma < 1$). In particular, Assumption 1 guarantees that $\delta^* > 0$ provided that the substitution between intertemporal consumption is elastic. Assumption 2 assures that sustainability, as defined by Pearce and Turner (90 a) and many others, will hold (i. e. $\gamma_u > 0$) along the balanced growth path, provided it exists. Finaly, Assumption 3 assures that the strong sustainability condition is satisfied (i. e $\gamma_n > 0$) either when the substitution between intertemporal consumption falls short or exceeds of unity.

3 The balanced growth path

The (optimal) balanced growth path, is defined by the paths of consumption and of the stock of natural resources, $\{\{\overline{C}(t)\}_{t=0}^{\infty}, \{\overline{N}(t)\}_{t=0}^{\infty}\}$, where $\overline{C}(t) = \overline{c}e^{\gamma t}$ and $\overline{N}(t) = \overline{n}e^{\gamma t}$, such that the endogenous growth rate γ and $\overline{c}$ and $\overline{n}$ are jointly determined from the steady state solution of the problem for the centralized economy.

Given the curvature properties of the utility function and of the equation for the accumulation of the natural resource the first order conditions are both necessary and sufficient.

The current value Hamiltonian is

$$H(c, n, q) = u(c, n) + q((\mu - \gamma_n)n - \alpha c),$$

and the first order conditions are

$$u_c(c^*(t), n(t)) = (c^*(t))^{-\sigma} n(t)^{\varphi(1-\sigma)} = \alpha q(t)$$

$$\dot{q}(t) = (\delta^* - \mu^*)q(t) - u_n(c^*(t), n(t))$$

for every admissible trajectories verifying equation (3) and the initial condition and the transversality condition

$$\lim_{t\to\infty} e^{-\delta^* t} q(t)n(t) = 0.$$

Proposition 1 *If assumption 1 , 2 and 3 holds then the long-run endogenous growth rate for the natural resource is*

$$\overline{\gamma}_n = \frac{1}{\sigma}\left(\frac{(1+\varphi)\mu + (1-\sigma)\gamma_a - \delta}{1+\varphi}\right) \tag{9}$$

such that

$$0 < \overline{\gamma}_n < \mu,$$

and the steady state values for the detrended variables verify

$$\frac{\overline{c}}{\overline{n}} = A(0)\left(\mu - \overline{\gamma}_n\right) \tag{10}$$

Proof. The modified Hamiltonian dynamic system may be written as

$$\dot{q}(t) = \varphi\Gamma(q(t), n(t)) \tag{11}$$

$$\dot{n}(t) = \Gamma(q(t), n(t)) \tag{12}$$

where

$$\Gamma \equiv \overline{\mu} - \alpha^{1-\frac{1}{\sigma}} q(t)^{-\frac{1}{\sigma}} n(t)^{\frac{\varphi(1-\sigma)}{\sigma}-1} = \overline{\mu} - \alpha \frac{c(t)}{n(t)}.$$

Therefore, we get $\frac{d\ln(q(t))}{dt} = \varphi \frac{d\ln(q(n))}{dt}$, which means that the dynamics of the system will tend to be degenerate.

Let $z \equiv \frac{c}{n}$, then we easily get

$$\dot{z}(t) = \alpha(1+\varphi)(z(t) - \overline{z})z(t) \tag{13}$$

where

$$\overline{z} = \frac{\overline{\mu}}{\alpha} = \frac{A(0)}{1+\varphi} \left[\delta - (1-\sigma)(\gamma_a + (1+\varphi)\mu\right] \tag{14}$$

Therefore

$$\gamma_a < \gamma < \gamma_a + \mu \tag{15}$$

As expected and as it is common in the literature, the endogenous growth rate of the economy in equation 9 depends on the preference, technological and natural parameters (see Smulders 2000, Rosenthal 1996). However, this rate is also the long run equilibrium growth rate of the natural asset. That is, given that natural resources are essential for production and in order to sustain a growing level of production and wellbeing, it becomes essential to assure a continuous flow of material and energy s to the economy, which, in turns depends on the natural dynamics of natural assets.

On the other hand, this positive endogenous growth rate will need to be lower than the maximal potential rate of regeneration of the environment, μ. This "upper limit" for the endogenous growth rate is needed in order to guarantee that the long run equilibrium path for the economy will accomplish the strong sustainability condition, thereby preventing the total depletion of natural resources and/or its ability to supply a continuous flow of matter and energy for production and consumption.

Moreover, the endogenous growth rate will lie between the exogenous growth rate of technological innovation and the sum of this rate with the environment regeneration rate, as showed in 15gnbgp. Again, economic growth is fueled by both the (exogenous) creation of new ideas and also for the human kind's ability to use natural resources in a way that allows a continuous and permanent supply of matter and energy provided by natural resources.

4 Dynamics

(From now on we will consider the optimal growth rates and will delete the overline notation.)

Proposition 2 *There are no transitional dynamics,*

$$C(t) = \overline{C}(t) = \overline{z}n(0)e^{\gamma t} \tag{16}$$

$$N(t) = \overline{N}(t) = n(0)e^{\gamma_n t} \tag{17}$$

Proof. Then the general solution for $z(t)$ is

$$\begin{aligned} z(t) &= \overline{z}\left(1+k_z\overline{z}e^{\alpha(1+\varphi)\overline{z}t}\right)^{-1} \\ &= \overline{z}\left(1+k_z\overline{z}e^{\delta^* t}\right)^{-1}, \end{aligned} \tag{18}$$

where k_z is a constant of integration. We will determine k_z such that the transversality condition holds. But as

$$\dot{n}(t) = n(t)(\mu^* - \alpha z(t))$$

then the solution for $n(t)$ becomes

$$\begin{aligned} n(t) &= k_n e^{\int_0^t(\mu^*-\alpha z(s))ds} = \\ &= k_n e^{\mu^* t-\alpha\overline{z}\left[\left(s-\frac{1}{\delta^*}\ln\left(1+\overline{z}k_z e^{\delta^* s}\right)\right)|_0^t\right]} = \\ &= k_n\left(\frac{1+\overline{z}k_z e^{\delta^* t}}{1+\overline{z}k_z}\right)^{\frac{1}{1+\varphi}}. \end{aligned}$$

We can determine the constant of integration k_n by using the data on n at time $t=0$. Then we get $w(0) = k_n = w_0$ which is given. Therefore

$$\begin{aligned} \lim_{t\to\infty} e^{-\delta^* t}q(t)n(t) &= \lim_{t\to\infty} e^{-\delta^* t}\alpha^{-1}z(t)^{-\sigma}n(t)^{(1+\varphi)(1-\sigma)} = \\ &= \lim_{t\to\infty} \alpha^{-1}\overline{z}^{-\sigma}n(0)^{(1+\varphi)(1-\sigma)}e^{-\delta^* t}\left(1+k_z\overline{z}e^{\delta^* t}\right)^{\sigma}\left(\frac{1+k_z\overline{z}e^{\delta^* t}}{1+k_z\overline{z}}\right)^{1-\sigma} = \\ &= \lim_{t\to\infty} e^{-\delta^* t}\left(1+k_z\overline{z}e^{\delta^* t}\right) = \\ &= \lim_{t\to\infty}\left(e^{-\delta^* t}+k_z\overline{z}\right) = \\ &= k_z\overline{z} \end{aligned}$$

which is equal to 0 if $k_z = 0$. Therefore we get $z(t) = \overline{z}$ and $n(t) = n_0$ as the solutions for the centralized model.

5 Effect of productivity change γ_A

Consider the case where there are changes on productivity

If $0 < \sigma < 1$, $\varphi > 0$ and $\gamma_A > \mu$ then

$$\frac{\partial}{\partial \gamma_A}\left[\frac{1}{\sigma}\left(\frac{(1+\varphi)\mu + (1-\sigma)\gamma_A - \delta}{1+\varphi}\right)\right] > 0 \tag{19}$$

That is, as expected, a raise in productivity will raise the rate of economic growth.

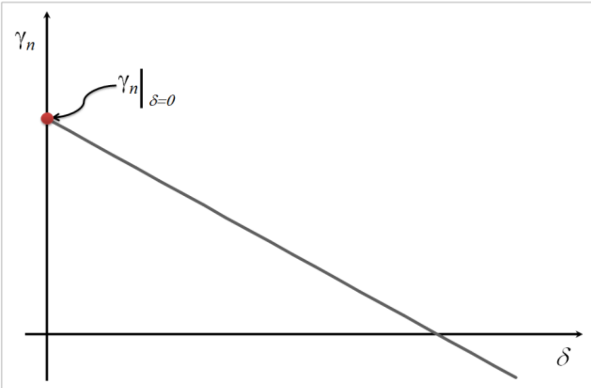
6 Effect of changes in preferences

First consider the case of changes in the rate of time preference, δ.

$$\frac{\partial \gamma_n}{\partial \delta} - \frac{1}{\sigma(1+\varphi)} < 0 \tag{20}$$

As expected, the balanced growth rate is inversely related with the rate of time preference. Recall that a positive value of this parameter means that well-being is less valued as later he is received. Therefore, higher values of δ decreases de willingness to save which then implies a lower rate of the balanced growth rate of the economy. This result is also consistent with the canonical literature of renewable resources where a less impatient society (lower value of the rate of time preference) will tend to save its endowments of renewable resources. Conversely, the higher the discount rate the faster the resources are likely to be depleted and, of course, the less of them will be available for future generations.

Figure 1 depicts the relationship between these two parameters when associated with an positive values of both σ and φ. As expected, the $\gamma_n - locus$ is downward sloping.



The higher the discount rate the lower the importance attached to future and hence the less likely society is to honor the idea of conserving its endowments of renewable resources. As a consequence the balanced growth rate of the economy will tend to be lower

We can thus state the following proposition:

Proposition 1: *an increase in the rate of time preferences will result in a lower endogenous growth rate for positive values of either the elasticity of intertemporal substitution of consumption* $(1/\sigma)$ *and the importance attached to natural resources into well-being,* φ.

1. Consider now the effects of changes of the elasticity of intertemporal substitution of consumption on the balanced growth rate γ_n.

$$\frac{\partial \gamma_n}{\partial \sigma} = -\frac{1}{\sigma}\left(\frac{\gamma_a}{1+\varphi} + \gamma_n\right)\left\{\begin{array}{c} > \\ = \\ < \end{array}\right\} 0 \quad iff \quad \gamma_a \left\{\begin{array}{c} < \\ = \\ > \end{array}\right\} - (1+\varphi)\gamma_n \tag{21}$$

Equation 21 tell us that the impact of the coefficient of risk aversion on the endogenous growth rate depends upon the relation between the rate of technological progress and the endogenous growth rate of the economy. From the sustainability criterions given by assumptions 1 and 2 it is clear that the coefficient of risk aversion (or the reciprocal of the elasticity of intertemporal substitution of consumption, $\theta = \frac{1}{\sigma}$) has a non-ambiguous effect over the balanced growth rate: higher values for the coefficient of risk aversion implies lower values for the sustainable endogenous grow rate. Recall that σ determines the household's willingness to shift consumption between different periods: the smaller is σ, the more slowly marginal utility falls as consumption raises and so the more willing household is to allow its consumption to vary over time. So for low levels of risk aversion consumers tend to raise their willingness to save, thereby leading to an increase in current investment. Accordingly, a lower growth rate will prevail.

Conversely, low values of the elasticity of intertemporal substitution of consumption,will decrease consumers' willingness to save which thereby implies a lower balanced growth rate. We can thus state the following proposition:

Proposition 2: *A decrease/raise of the coefficient of risk aversion will result in a higher/smaller endogenous growth rate of the economy.*

7 Conclusions

In this paper we study a simple endogenous growth model in which the two engines of growth are the exogenous technical progress in dematerialization and the accumulation of a renewable natural resource. The model is also labeled as been "endogenous" as the rate of growth of natural capital is endogenously determined and should lie between zero and the rate of technical progress. We assume that new technologies generates a process of dematerialization, i.e., the possibility of producing more with a decreasing use of raw materials. In this context, it is possible to combine permanent economic growth with permanent growth of the environmental asset.

This growth rate is also the long run growth rate of economy. Given that natural resources are essential for production and in order to sustain a growing level of production and wellbeing, it becomes essential to assure a continuous flow of material and energy s to the economy, which, in turns depends on the natural dynamics of natural assets.

This positive endogenous growth rate will need to be lower than the maximal potential rate of regeneration of the environment. This "upper limit" for the endogenous growth rate is needed in order to guarantee that the long run equilibrium path for the economy will accomplish the strong sustainability condition, thereby preventing the total depletion of natural resources and/or its ability to supply a continuous flow of matter and energy for production and consumption. Moreover, the endogenous growth rate will lie between the exogenous growth rate of technological innovation and the sum of this rate with the environment regeneration rate. Therefore, even in the case in which the physical rate of renewal is small, this will allow for unbounded growth. Finaly, in our simple model, there is no transitional dynamics.

There are several extensions that this paper suggests. An obvious one is to consider the accumulation of the man-made capital and as a result a modified production function. For its simplicity, one possible candidate is the well known AK technology.

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