Nearest Neighbor Connectivity in Two-Dimensional Multihop MANETs

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Abstract A Mobile Ad Hoc Network (MANET) is characterized to be a network with free, cooperative, and dynamic nodes, self-organized in a random topology, without any kind of infrastructure, where the communication between two nodes usually occurs using multihop paths. The number of hops used in the multihop path is an important metric for the design and performance analysis of routing protocols in MANETs. In this paper, we derive the probability distribution of the hop count of a multihop path between a source node and a destination node, fixed at a known distance from each other, and when a fixed number of nodes are uniformly distributed in a region of interest. This distribution is obtained by the Poisson randomization method. To obtain the multihop path, we propose a novel routing model in which the nearest distance routing protocol is analyzed. Numerical results are obtained to evaluate the performance of the nearest distance routing protocol.

1 Introduction

When the source and destination nodes of a MANET are at a distance greater than the transmission range, the communication between them is made via a multiple hop path that is determined by the routing protocol (cf., e.g., [8]). One of the most popular strategies a node can use to decide to which neighbor node it should forward a given packet is the nearest distance routing protocol (NR), for which the packet is forwarded to the nearest relay node in the direction of the destination node.

As stated in [7] and references therein, one of the most important metrics to evaluate the performance of routing protocols is the number of hops of the multihop path. In [1], we have derived the hop count distribution for the one-dimensional

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scenario with relay nodes uniformly distributed between the source and destination nodes. However, the derivation of the hop count distribution in a two-dimensional scenario must take into account, among other factors, the transmission range and the routing protocol, aside from the node spatial distribution. The interaction of these characteristics turns the derivation of the hop count distribution a difficult task. This is the reason why, despite its importance, there are few analytical studies on the subject and most of them just consider single link models (cf. [5] and [10]) and/or approximation results (cf. [4] and [7]).

In [5] relay nodes are assumed to be distributed according to a Poisson process and the distribution of the distance from the source to the furthest neighbor node within transmission range is derived. The analysis was extended in [10] to a model where a finite number of relay nodes are uniformly distributed in a region of interest, but again only assuming a single link model. Few papers focus their analysis in more than a single link. In [4], an approximation for the relationship between the number of hops and the distance between the source and the destination nodes is derived, and an approximation for the probability of existence of a multihop path between the source and destination nodes is derived in [7].

In [9] one of the few closed-form results on the hop count distribution is derived for the case in which nodes are randomly distributed according to a Poisson process, for both one-dimensional and two-dimensional networks, and using three routing protocols: the nearest, the furthest and the random routing protocol. However, the average hop length has to be used and estimated, turning the obtained results approximations of the exact hop count distribution.

In this paper, we derive the exact hop count probability distribution with an arbitrary number of hops, when the source and destination nodes are fixed, at a known distance form each other, and a known and fixed number of relay nodes are uniformly distributed in a region of interest. To obtain the multihop path, we propose a novel propagation model where the routing region of each relay node is defined by a given angular span and a radius equal to the transmission range. Since the angular span depends on the distance between the emitter and destination nodes, we call this model the dynamic propagation model,. Inside each routing region, we use the NR protocol to choose the relay node to forward the packet.

The mathematical analysis of the problem of an existing path on a random set of points, with the source and destination nodes at known locations, is often called a navigation problem. Within this literature, the paper [2] proposes a model with the nearest routing protocol using routing regions with a fixed angular span. The authors proved that when the number of random nodes is large enough, almost surely exists a path between the source and the destination nodes.

As far as we know, our results are the first exact analytical results for the hop count distribution with an arbitrary number of hops in a two-dimensional scenario, when a finite number of relay nodes are uniformly distributed in an area of interest. These results are suitable to use when the number of hops is not too large, because the dynamic angular span decreases when the source or relay nodes are far way from the destination node. However, in MANETs the number of hops between the source and destination nodes cannot be large due to the small duration of multihop paths with a large number of hops [6]. In dense networks that does not constitute a problem since the multihop path is similar to a path on a straight line. Note that the usage of the position-based protocols requires that a node knows its own geographical position and the geographical position of the destination node, but the localization problem of the nodes are not focused in this paper. We also should note that we consider the transmission range of each node constant, not taking into account the SINR (signal-to-interference noise ratio), which will be the scope of future research.

The outline of this paper is the following. In Section 2 we describe the dynamic propagation model. In Section 3 we derive the hop count distribution for the NR protocol. In Section 4 we present some numerical results to evaluate the performance of the NR protocol. Finally, in Section 5, we conclude the paper.

2 Model Description

We consider an ad hoc network with the source node fixed at the origin and the destination node fixed at a distance L from the source node. A multihop path with m hops is defined as an existing path from the source to the destination node using exactly m relay nodes. Denote by $X_i, 1 \leq i \leq m$, the location of the relay node i of a multihop path, with these nodes ordered according to their distance to the origin, and let $X_0 = (0,0)$ and $X_{m+1} = (L,0)$ denote the locations of the source and destination nodes, respectively. Note that, without loss of generality, we have assumed that the destination node is located in the x-axis. Given a fixed transmission range R, $0 < R < L$, equal for all nodes, nodes i and j are connected with zero hops if $\|X_i - X_j\| < R$.

We assume that the locations of the source node, the destination node, and all relay nodes of the multihop path belong to a compact set $\Omega \subset \mathbb{R}^2$, with area B. The set Ω is defined by an isosceles triangle with one vertice at the origin $(0,0)$ with associated angle $\phi_0 = 2\arctan(R/L)$, and the height of the triangle lies on the horizontal axis and is equal to L. The definition of the set Ω is needed to avoid analytical intractability and preclude that a given multihop path loops around the destination, see [8]. For efficient routing progress towards the destination, we consider that each relay node transmits within a routing region limited by the transmission radius R and an angular span oriented to the destination node. The angular span ϕ_i of relay node i is chosen in a dynamic way, being dependent on the location X_i of the relay node, and is such that it originates a triangle with vertices at points (L,R), $(L,-R)$ and X_i, increasing when it gets closer to the destination node and decreasing when the relay node gets further away from the destination node. This is the reason why we denominate the model as the dynamic propagation model. Within each routing region the relay node chosen to forward the packet will be the nearest relay node from the emitter node. The polar coordinates of the location of relay node i relative to the location of relay node $i-1$ are denoted by (r_i, θ_i), assuming that $-\pi \leq \theta_i \leq \pi$. In Figure 1 we can observe a multihop path with 3 hops using the NR protocol and the dynamic propagation model. Note that if a given node is in the range of the destination node, they will connect directly.

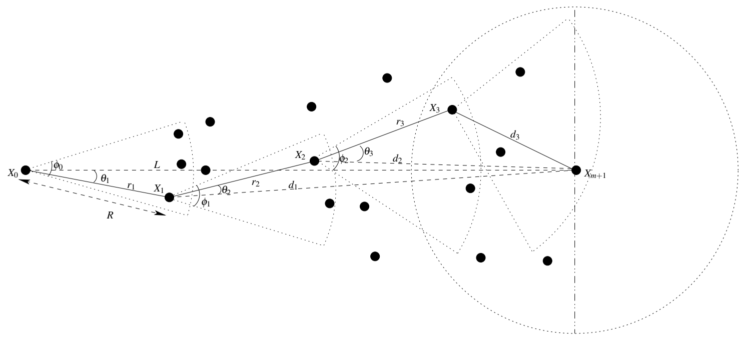

Fig. 1 Dynamic propagation model with the NR protocol for a path with 3 hops.

3 Hop Count Distribution

To describe the routing regions of each relay node, we make a translation and rotation of the plane to locate the origin of the new plane at the current emitter node (in this case at relay node i), with horizontal axis being the line drawn from the emitter node to the destination node. For a relay node i located at X_i, the routing region relative to X_i is denoted by $\mathscr{A}_i \equiv \mathscr{A}(X_i, X_{m+1}, \phi_i)$ and, at each hop, an angular slice of a circular disk with radius R and with area $\frac{\phi_i}{2}R^2$ is covered (see Figure 2). More precisely, the routing region of relay node i relative to X_i is defined by

$$\mathscr{A}_i \equiv \mathscr{A}(X_i, X_{m+1}, \phi_i) = \left\{(r,\theta) : 0 < r < R, -\phi_i^- \leq \theta \leq \phi_i^+\right\}.$$

The angular span ϕ_i is dynamic and depends of the location of the relay node. Given (r_i, θ_i) and the distance from relay node $i-1$ to the destination node, d_{i-1}, the distance from relay node i to the destination node, d_i, is given by the function

$$d_i \equiv f(d_{i-1}, r_i, \theta_i) = \sqrt{(d_{i-1} - r_i\cos\theta_i)^2 + (r_i \sin\theta_i)^2}, \qquad 1 \leq i \leq m,$$

with $d_0 = L$. The angle ϕ_i of relay node i can then be written as a function of d_{i-1} and (r_i, θ_i), $\phi_i \equiv \phi(d_{i-1}, r_i, \theta_i)$, and is given by

$$\phi_i = \arcsin\left(\frac{R - sign(\theta_i) b_i}{s_i^+}\right) + \arcsin\left(\frac{R + sign(\theta_i) b_i}{s_i^-}\right),$$

where $b_i = r_i \sin\theta_i$, so that $|b_i|$ is the minimum distance between X_i and the axis that goes from X_{i-1} to X_{m+1}, and $s_i^{\pm} = \sqrt{(d_{i-1} - r_i\cos\theta_i)^2 + (R \mp sign(\theta_i)b_i)^2}$ is the distance between X_i and $(L, \pm R)$; see Figure 2. Using geometric arguments, we can show that $\phi_i = \phi_i^+ + \phi_i^-$, where ϕ_i^+ is the angle formed by the points (L,R), X_i and X_{m+1}, being given by $\phi_i^+ = \arcsin\left(\frac{R - sign(\theta_i)b_i}{s_i^+}\right) + sign(\theta_i)\arcsin\left(\frac{b_i}{d_i}\right)$, and ϕ_i^- is the angle formed by the points $(L,-R)$, X_i and X_{m+1}, being given by $\phi_i^- = \arcsin\left(\frac{R + sign(\theta_i)b_i}{s_i^-}\right) - sign(\theta_i)\arcsin\left(\frac{b_i}{d_i}\right)$.

Denote by $\mathscr{V}_i$ the vacant region of relay node i, defined to be the subset of the routing region of relay node i that has no relay nodes. That is, since the relay node selected is the closest one from the emitter node, the vacant region of relay node i

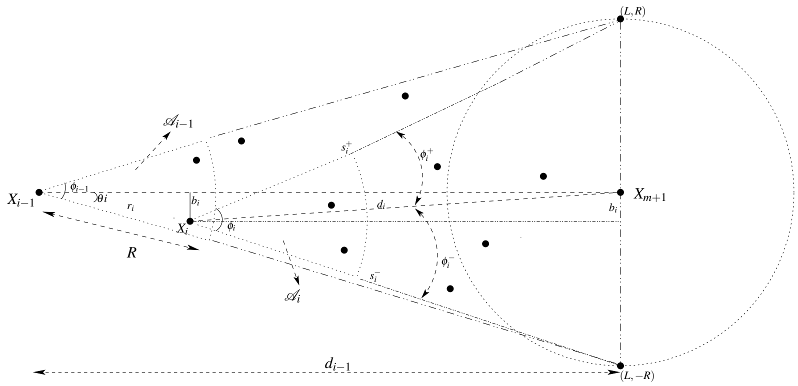

Fig. 2 Routing regions and angular spans of relay nodes $i-1$ and i.

is given by the set of points that are closer to i than relay node $i+1$, having an area $V_i = \frac{\phi_i}{2} r_{i+1}^2$; see Figure 3.

The hop count probability distribution is obtained by using Poisson randomization, [3], consisting in randomizing the number of relay nodes by assuming that relay nodes are distributed in Ω according to a Poisson process with rate λ. A precise argument for the spatial Markov property in more general spaces can be found in [11]. By conditioning in the number of relay nodes that lie in Ω, the results for the case in which a fixed and known number of relay nodes are uniformly distributed in Ω pops up. Denote by $\mathbf{l}_m = (l_1, l_2, \ldots, l_m)$ the vector of relative locations of the m relay nodes, with $l_i = (r_i, \theta_i)$, and let $\mathrm{d}\mathbf{l}_m = \mathrm{d}\theta_m \mathrm{d}r_m \mathrm{d}\theta_{m-1} \mathrm{d}r_{m-1} \ldots \mathrm{d}\theta_1 \mathrm{d}r_1$. Recall that B denotes the area of Ω.

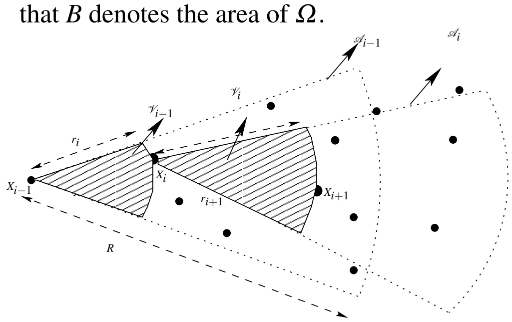

Fig. 3 Routing regions and vacant regions of relay nodes $i-1$ and i.

Theorem 1. *Given that there are n relay nodes uniformly distributed on Ω, the probability that the hop count is equal to m, for a multihop path selected by the dynamic propagation model with the NR protocol, is given by*

$$P(M = m | N = n) = \int_{N_m} \frac{n!}{(n-m)!} \frac{1}{B^m} \left(1 - \frac{1}{B} \sum_{i=0}^{m-1} V_i \right)^{n-m} \prod_{i=1}^{m} r_i \;\; \mathrm{d}\mathbf{l}_m \qquad (1)$$

with $K \le m \le n$ and $N_m = \left\{ \mathbf{l}_m : l_i = (r_i, \theta_i) \in \mathscr{A}_{i-1}, i = 1, 2, \ldots, m, d_m < R \le d_{m-1} \right\}$.

Proof. We first derive the joint location density of the m relay nodes of the multihop path. For that, fix $(r_1,\theta_1) \in \mathscr{A}_0 = \left\{(r_1',\theta_1') : 0 < r_1' < R, -\frac{\phi_0}{2} < \theta_1' < \frac{\phi_0}{2}\right\}$ and define $\mathscr{V}_0 = \left\{(r_1',\theta_1') : 0 < r_1' < r_1, -\frac{\phi_0}{2} < \theta_1' < \frac{\phi_0}{2}\right\}$ and $\mathscr{V}_0^{\varepsilon} = \{(r_1',\theta_1') : r_1 \le r_1' < r_1+\varepsilon_1, \theta_1 \le \theta_1' < \theta_1+\varepsilon_2\}$. Denote by $N(A)$ the number of points of the Poisson process in A. By the independent increment property of a Poisson process, we have

$$\begin{aligned} P(N(\mathscr{V}_0)=0, N(\mathscr{V}_0^{\varepsilon})>0) &= P(N(\mathscr{V}_0)=0)\,P(N(\mathscr{V}_0^{\varepsilon})>0) \\ &= \mathrm{e}^{-\lambda\frac{\phi_0}{2}r_1^2}\left(1-\exp\left(-\lambda\int_{r_1}^{r_1+\varepsilon_1}\int_{\theta_1}^{\theta_1+\varepsilon_2} r\,\mathrm{d}r\,\mathrm{d}\theta\right)\right) \\ &= \mathrm{e}^{-\lambda\frac{\phi_0}{2}r_1^2}\lambda\int_{r_1}^{r_1+\varepsilon_1}\int_{\theta_1}^{\theta_1+\varepsilon_2} r\,\mathrm{d}r\,\mathrm{d}\theta + o(\varepsilon_1\varepsilon_2). \end{aligned}$$

The density of the location of the first relay node being at (r_1,θ_1) is given by $h(r_1,\theta_1) = \lim_{\varepsilon_1,\varepsilon_2\to 0^+}\frac{P\left(N(\mathscr{V}_0)=0,N(\mathscr{V}_0^{\varepsilon})>0\right)}{\varepsilon_1\varepsilon_2} = \lambda r_1 \mathrm{e}^{-\lambda\frac{\phi_0}{2}r_1^2}$.

To derive the density location of the first two relay nodes, we make a rotation and translation of the plane in order to place the origin of the new plane at $(r_1+\varepsilon,\theta_1)$ with horizontal axis being the line drawn from $(r_1+\varepsilon,\theta_1)$ to the destination node. Proceeding in a similar way to the one used to derive the density of the location of the first relay node, one may conclude (see [6]) that the density of the locations of the first two relay nodes being (r_1,θ_1) and (r_2,θ_2) is $h(r_1,\theta_1,r_2,\theta_2) = \lambda^2 r_1 r_2 \mathrm{e}^{-\lambda\frac{\phi_0}{2}r_1^2}\mathrm{e}^{-\lambda\frac{\phi_1}{2}r_2^2}$.

Proceeding in the same manner until the m-th relay node is connected with no hops with the destination node, we obtain the joint density of the locations of the m relay nodes of the multihop path, $h(\mathbf{l}_m) = \lambda^m \mathrm{e}^{-\lambda\sum_{i=1}^{m}\frac{\phi_{i-1}}{2}r_i^2}\prod_{i=1}^{m} r_i$, where the node locations are in N_m and the last relay node is m because $d_m < R \le d_{m-1}$. Integrating $h(\mathbf{l}_m)$ over the set $N_m = \{\mathbf{l}_m : l_i = (r_i,\theta_i) \in \mathscr{A}_{i-1}, i=1,2,\ldots,m, d_m < R \le d_{m-1}\}$, we obtain the probability that the hop count is m for the nearest distance routing protocol, when the relay nodes are randomly distributed according to a Poisson process:

$$P(M=m) = \int_{N_m} \lambda^m \mathrm{e}^{-\lambda\sum_{i=1}^{m}\frac{\phi_{i-1}}{2}r_i^2}\prod_{i=1}^{m}\ \mathrm{d}\mathbf{l}_m. \tag{2}$$

Multiplying equation (2) by $\mathrm{e}^{\lambda B}$, where B is the area of Ω, we obtain

$$\begin{aligned} \mathrm{e}^{\lambda B}P(M=m) &= \mathrm{e}^{\lambda B}\int_{N_m}\lambda^m \mathrm{e}^{-\lambda\sum_{i=0}^{m-1}V_i}\prod_{i=1}^{m} r_i\ \mathrm{d}\mathbf{l}_m \\ &= \int_{N_m}\lambda^m\sum_{n=0}^{\infty}\frac{(\lambda B)^n}{n!}\left(1-\frac{1}{B}\sum_{i=0}^{m-1}V_i\right)^n\prod_{i=1}^{m} r_i\ \mathrm{d}\mathbf{l}_m \\ &= \sum_{n=m}^{\infty}\frac{(\lambda B)^n}{n!}\int_{N_m}\frac{n!}{(n-m)!}\frac{1}{B^m}\left(1-\frac{1}{B}\sum_{i=0}^{m-1}V_i\right)^{n-m}\prod_{i=1}^{m} r_i\ \mathrm{d}\mathbf{l}_m \end{aligned}$$

where the change between the sum and the integral follows by the dominated convergence theorem. On the other hand, conditioning on the value of N, which is Poisson distributed with mean λB, by the total probability law $e^{\lambda B}P(M=m) = \sum_{n=m}^{\infty} P(M=m|N=n)\frac{(\lambda B)^n}{n!}$. Since the coefficients of $\frac{(\lambda B)^n}{n!}$ in the previous two expressions for $e^{\lambda B}P(M=m)$ must match, the result follows. □

4 Numerical Results

In this section we evaluate the performance of the dynamic propagation model for the NR protocol. We scale all parameters with respect to the distance between the source and destination nodes assuming that $L=1$, leading the set Ω to have area $B = RL$. Therefore, depending on the value of R, for $1/(K+1) < R \leq 1/K$, $K \in \mathbb{N}$, we have multihop paths with a minimum number of hops equal to K. The results were obtained by numerical integration using a Monte Carlo algorithm. Despite the multidimensional integration, it is relatively simple and not too much time consuming the calculation over 6 hops, which is a very large number of hops for a MANET [6].

Figure 4 shows the connectivity probability with the minimum number of hops K, $K = 1,2,3$, with the NR protocol and for different values of the number of nodes. We can observe that when the number of nodes increases the minimum hop count probability decreases and approaches the value 0, and so the NR protocol is ineffective in a dense network because it cannot transmit with a high probability with the minimum number of hops. For the same number of relay nodes, the hop count probability with the minimum number of hops decreases as K increases.

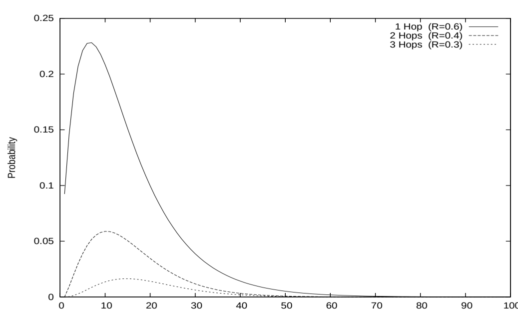

Fig. 4 Connectivity probability with the minimum number of hops.

In Figure 5, we obtain the hop count probability with different values of the number of hops. We consider $R = 0.3$, and $K = 3,4,5,6$, and observe that, when there is a small number of nodes, the NR protocol with $K+1 = 4$ hops has the highest probability, whereas when there is a large number of nodes, the hop count probability with $K+3 = 6$ has the highest probability. Again the probability with the minimum number of hops K with the NR protocol is very ineffective, since it has the smallest probability. Despite that, all probabilities ($K = 3,4,5,6$) approach zero with the increase of the number of nodes, and the probabilities obtained for paths with a large number of hops are generally larger than the ones obtained for paths with a smaller number of hops.

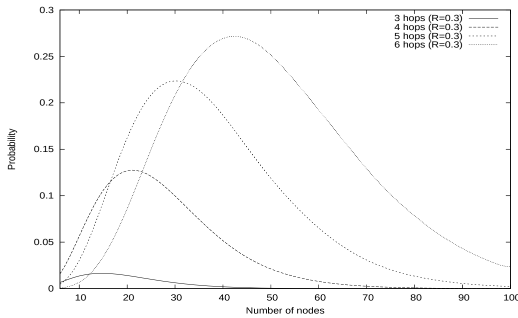

Fig. 5 Connectivity probability with hop count equal to $K = 3,4,5,6$.

5 Conclusion

In this paper we focused on the connectivity in two-dimensional wireless ad-hoc networks. We have assumed that the source and the destination nodes are fixed, at a known distance from each other, and that a fixed and known number of relay nodes are uniformly distributed in a region of interest. To find a multihop path, we proposed a novel model called the dynamic propagation model. Using this model, we derived the hop count probability distribution when the multihop path chosen follows the nearest distance routing protocol. As far as we know, these are the first exact analytical results for the hop count probability distribution. The numerical results derived allowed us to conclude that the NR protocol is not suitable for dense networks.

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