Green Measures for Nonlocal Diffusion Equations

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Abstract

Let $X(t), t \geq 0$ be a time homogeneous Markov process in $\mathbb{R}^d$ starting from the point $x \in \mathbb{R}^d$. For a function $f : \mathbb{R}^d \to \mathbb{R}$ we consider the following heuristic object

$$V(f,x) = \int_0^\infty \mathbb{E}^x[f(X(t)]\,\mathrm{d}t.$$

If this quantity exists, then $V(f,x)$ is called the *potential* for the function f. The notions of potentials is well known in probability theory, see e.g., [1, 4]. The existence of the potential $V(f,x)$ is a difficult question and the class of admissible f shall be analyzed for each process X separately. One approach is based on the use of the generator L of the process X. Namely, the potential $V(f,x)$ may be constructed as the solution to the following equation:

$$-LV = f.$$

In analogy with the PDE framework, we would like to have a representation

$$V(f,x) = \int_{\mathbb{R}^d} f(y)\mathcal{G}(x,\mathrm{d}y),$$

where $\mathcal{G}(x,dy)$ is a measure on $\mathbb{R}^d$. This measure is nothing but the fundamental solution to the considered equation and traditionally may be called the Green measure for the operator L.

In this talk we study the existence of Green measures for Markov processes with a nonlocal jump generator without second moment and a suitable condition on its Fourier transform. This talk is based, in particular, on the joint works [2, 3].

References

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[4] D. Revuz and M. Yor. *Continuous martingales and Brownian motion*, volume 293 of *Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences]*. Springer-Verlag, Berlin, 3rd edition, 1999.