Individual growth models with stochastic differential equations

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Abstract

To describe individual growth dynamics, regression methods are inappropriate. So, we use stochastic differential equation (SDE) models that, in the most general form, can be written as $dY(t) = \beta(\alpha - Y(t))dt + \sigma dW(t)$, where $Y(t) = h(X(t))$ with $X(t)$ being the weight of an animal at age t and h an appropriate strictly increasing C^1 function. The parameters are $\alpha = h(A)$, where A is the maturity weight of the animal, β the rate of approach to maturity and σ the intensity parameter of the random fluctuations. $W(t)$ is a standard Wiener process.

These more realistic models can help farmers optimize the profit obtained by raising and selling an animal. To that end, we obtain the expected value and the standard deviation of the profit as a function of the selling age under the more general and realistic market situation where the selling price per kg paid to farmers depends on the animal's age and weight category and feeding costs vary with the animal's weight. We apply results to real weight data of Mertolengo and Alentejana breeds cattle males [2].

We have used maximum likelihood theory to estimate the parameters. However, for cattle data, it is often not feasible to obtain animal's observations at equally spaced ages nor even at the same ages for different animals and there is typically a small number of observations at older ages. For these reasons, maximum likelihood estimates can be quite inaccurate, being interesting to consider in the likelihood function a weight function associated to the elapsed times between two consecutive observations of each animal, which results in the weighted maximum likelihood method. We compare the results obtained from both methods [1].

Additionally, since model parameters may vary from animal to animal and that variability can be partially explained by their genetic differences, we have extended the study to SDE mixed models, where the variation among animals of the parameters α, β or both is assumed to be random. For these three cases, the maximum likelihood estimation method was applied using a Delta method approximation to solve the integrals involved in the maximum likelihood function [3].

References

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