

Programa

09:00-09:10 *Abertura do Encontro*

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09:10-09:30 *Imme van Den Berg, Nam Van Tran*

On feasibility and robustness of flexible systems of linear equations.

09:30-09:50 *José Carmo (online)*

Modal logics and applications

09:50-10:10 *Marco Garapa*

On the Dynamics of Beliefs

10:10-10:30 *Helena Soares*

Quaternionic Numerical Range

10:30-10:50 *Rui Albuquerque*

On the volume of unit vector fields in 2 and 3 dimensions

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SDE harvesting models in random environments: The effect of Allee effects, model robust properties and profit optimization

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Semi-parametric estimation in Statistics of Extremes

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Individual growth models with stochastic differential equations

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Green Measures for Nonlocal Diffusion Equations

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Approximations to the Binomial distribution, its bounds, relative and absolute precision

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Impulsive coupled systems with functional boundary conditions

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On the Liapunov convexity theorem under pointwise constraints

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Constraint programming approach to multiobjective Forest Management with adjacency restrictions

15:30-15:50 *Marília Pires, Tomás Bodnár*

Artificial Stress Diffusion in Numerical Simulations of Viscoelastic Fluids Flows

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Dissipation

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16:40-17:00 *Luís Silva*

Period incrementing and Milnor attractors for non autonomous families of
at top tent maps

17:00-17:20 *Sara Perestrelo, Clara Grácio, Nuno Ribeiro, Luís Lopes*

A Multiscale Network with Percolation Model to Describe the Spreading of Forest Fires

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Flipped classroom as a mathematics learning space for students

17:40-17:50 *Encerramento*

On feasibility and robustness of flexible systems of linear equations

Imme van Den Berg, Nam Van Tran

Abstract

A *flexible system* $A|B$ is a system of linear equations in which each coefficient and each element of the right-hand side have an individual imprecision in terms of a scalar neutrix, which is a convex group of nonstandard real numbers. They can be solved by method consisting of (i) a LU-decomposition, (ii) assignment of parameters to the imprecisions, (iii) Gaussian elimination and (iv) substitution of the parameters into the solution by their range. We give conditions for the system to be feasible, i.e. the essential part of the upper triangular matrix LU-decomposition is obtained from A , while the constraints resulting from the imprecisions do only interfere with the range of the solutions. In the case of non-singular systems, we determine the maximal robustness matrix, i.e. the matrix E consisting of the maximal imprecisions such that $A|B$ and $(A+E)|B$ have the same solutions.

Modal logics and applications

José Carmo

Abstract

Propositional, normal and non-normal modal logics. Some ramifications and applications: belief logics, temporal logics, action logics and deontic logics, among others. Particular reference to deontic and action logics: some paradoxes and their use for the characterization of normative concepts and specification of the interaction between agents.

On the Dynamics of Beliefs

Marco Garapa

Abstract

In the AGM paradigm, an agent's belief state is represented by a logically closed set of sentences (also called belief set) and primacy is given to the new information, in the sense that the new information is always fully incorporated into the agent's belief state. Not long after its publication, several variants of that model started to appear in the literature. From among those proposals we highlight (for being the ones that are directly related to this lecture):

- (i) The use of sets of sentences not (necessarily) closed under logical consequence – the so-called belief bases – rather than belief sets to represent the belief states of an agent;
- (ii) Classes of, so-called, non-prioritized operators, which are operators that do not satisfy the success postulate.

In this talk we present some of the models of belief change as well as the intuitions and motivations underlying them, and mention some of the recent and ongoing work of our group in this area.

Keywords: Belief Change, Non-prioritized Belief Change, Belief Bases, Profile Dynamics.

Quaternionic Numerical Range

Helena Soares

Abstract

Let T be a bounded linear operator on the complex Hilbert space \mathcal{H} with inner product $\langle \cdot, \cdot \rangle$. The numerical range of T is the set in the complex plane

$$W(T) = \{ \langle Tx, x \rangle : \|x\| = 1, x \in \mathcal{H} \}.$$

Thus $W(T)$ is simply the range of the unit circle of H under the quadratic map $x \mapsto \langle Tx, x \rangle$ induced by T .

One motivation in studying the numerical range of T is that it gives an estimate of the location of the spectrum $\sigma(T)$, the set of scalars λ for which $T - \lambda I$ is not invertible. In fact, $\sigma(T) \subset \overline{W(T)}$.

Over the years, the investigation of the numerical range continuously increased, including linear operators on infinite-dimensional complex Hilbert spaces and complex Banach spaces. In a different direction, Kippenhahn [Ki] introduced in 1951 the study of the numerical range for quaternionic operators over finite-dimensional Hilbert spaces.

A striking difference between the quaternionic and complex numerical ranges is convexity. Contrary to what happens in the complex case, the quaternionic numerical range is not convex in general. The failure of convexity occurs soon for 1×1 quaternionic matrices. Moreover, for a given finite operator it is very difficult to characterize its numerical range whose shape is, for the most part, unknown.

There is a substantial body of work on the subject of numerical range for operators on complex Hilbert spaces, both in finite and infinite dimension. Much different is the case of quaternionic linear operators. While earlier studies on the quaternionic numerical range have been carried out in finite dimension by the work of Kippenhahn [9], Au-Yeung [12], So and Thompson [11] and, more recently, Kumar [9] and some researchers in this team [1]-[6], there is a lack of results in infinite-dimensional quaternionic Hilbert spaces. A possible explanation for this is the nonexistence, until recently, of a suitable notion of spectrum in the quaternionic setting. In 2007, Colombo, Gentili, Sabadini and Struppa [7] proposed the notion of S-spectrum, also known by spherical spectrum, which lead to spectral theorems for quaternionic normal operators, see [10].

Together with my colleagues Cristina Diogo, Luís Carvalho e Sérgio Mendes, we are studying the numerical range of bounded linear operators on quaternionic Hilbert spaces. In my talk I will present some of the problems we have addressed and results we have so far obtained. Also, I would like to refer to some future. In particular, we would like

to find out if an analogue of Anderson's theorem in the complex setting (on conditions for the numerical range of a finite matrix to be equal to the unit circular disc) by means of algebraic geometry tools is possible in the quaternionic setting.

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On the volume of unit vector fields in 2 and 3 dimensions

Rui Albuquerque

Abstract

Starting from a well-known idea of the proof via calibrations that the Hopf vector fields are unique minimizers of the functional volume of vector fields on the sphere S^3 , we show the possibility of using a new differential system on the tangent sphere bundles either of a Riemann surface (dim 2) or of a space of dim 3 of constant sectional curvature. New calibrations give new results of the same sort.

SDE harvesting models in random environments: The effect of Allee effects, model robust properties and profit optimization

Carlos A. Braumann, Nuno M. Brites, Clara Carlos

Abstract

We seek model robust properties, so we work with a general function $f \in C^1 :]0, +\infty[\rightarrow \mathbb{R}$, which, besides mild technical assumptions (like having left limit $f(0^+)$ and $\lim_{X \rightarrow 0^+} Xf(X) = 0$), satisfies only qualitative conditions dictated by biological considerations. Since one main purpose is to study the effect of Allee effects, we compare general models without Allee effects (f strictly decreasing with $f(0^+) > 0$ and $f(+\infty) < 0$, which implies the existence of a carrying capacity $K > 0$ such that $f(K) = 0$) with general models with Allee effects (there is an $L \in]0, K[$ such that $f(L) > 0$, $f(K) = 0$, $f(X)$ decreases for $X > L$, but, due to the Allee effects, $f(X)$ increases for $0 < X < L$).

Considering autonomous efforts $E(t) = E(X(t))$ and defining the **geometric average net growth rate** $n(X) = g(X) - qE(X)$ (difference between the geometric average natural growth rate $g(X) = f(X) - \sigma^2/2$ and the harvesting mortality rate $qE(X)$), the deciding factor is the sign of its limit $n(0^+)$ at low population sizes. If the sign is negative (overfishing), we have population extinction. If the sign is positive, we have sustainability, with $X(t)$ ergodic converging as $t \rightarrow +\infty$ to a stochastic equilibrium with a stationary density function proportional to the speed density. That was shown in [1, 2, 3] for general models without Allee effects and in [13] for general Allee effects models without harvesting (i.e. with $E(x) \equiv 0$). We have now proved it for general Allee effects models with autonomous harvesting.

However, realistic extinction (in the sense of population dropping below some small positive extinction threshold $a < X(0)$) occurs in both cases. Expressions for the mean and standard deviation of extinction times can be seen in [12, 4] and [11] uses them to determine the influence of Allee effects on extinction.

When applying to real data to numerically compare harvesting policies and their associated profits, we do obviously work with specific models, namely the logistic model $f(X) = r(1 - \frac{X}{K})$ ($r > 0$), which has no Allee effects, and the logistic-like model with Allee effects $f(X) = r(1 - \frac{X}{K})(\frac{X-A}{K-A})$ (with $A \in]-K, 0[$, i.e. with weak Allee effects since strong Allee effects lead to extinction, even without harvesting).

In previous work [5], for the logistic model and using data from [14] on the Pacific halibut (*Hippoglossus hippoglossus*), we have shown that the harvesting policy with variable effort is inapplicable, whereas the optimal harvesting policy with constant effort $E(t) \equiv E$ is easily applicable and leads to population sustainability, with only a slightly lower profit. So, we have also considered [6, 7, 9] stepwise policies, which are applicable but share some of the problems of the optimal variable effort policy, penalized profit optimal policies (with an artificial running energy cost on the effort), which eliminate some of the disadvantages but are still inapplicable, and combinations of these policies.

We also applied these policies to the logistic-like model with Allee effects [8, 10] to study the influence of Allee effects and check whether they should be taken into account when designing harvesting policies.

Acknowledgments

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Semi-parametric estimation in Statistics of Extremes

Lígia Henriques-Rodrigues

Abstract

The main objective of the Extreme Value Theory is the prediction of rare events, i.e., potentially disastrous events, of enormous importance to society and of great social impact, so that the proper estimation of the parameters related to this type of events plays an important role in Statistics of Extremes. The distribution of the maximum of n independent and identically distributed observations, after adequate normalization, converges to the so-called Extreme Value distribution, where the shape parameter of the distribution, ξ , is called the tail index or extreme value index. In this talk, a brief overview of the semi-parametric inferential procedures used to estimate ξ and other parameters of rare events is presented.

Individual growth models with stochastic differential equations

Gonçalo Jacinto, Patricia Filipe, Carlos Braumann, Nelson Jamba

Abstract

To describe individual growth dynamics, regression methods are inappropriate. So, we use stochastic differential equation (SDE) models that, in the most general form, can be written as $dY(t) = \beta(\alpha - Y(t))dt + \sigma dW(t)$, where $Y(t) = h(X(t))$ with $X(t)$ being the weight of an animal at age t and h an appropriate strictly increasing C^1 function. The parameters are $\alpha = h(A)$, where A is the maturity weight of the animal, β the rate of approach to maturity and σ the intensity parameter of the random fluctuations. $W(t)$ is a standard Wiener process.

These more realistic models can help farmers optimize the profit obtained by raising and selling an animal. To that end, we obtain the expected value and the standard deviation of the profit as a function of the selling age under the more general and realistic market situation where the selling price per kg paid to farmers depends on the animal's age and weight category and feeding costs vary with the animal's weight. We apply results to real weight data of Mertolengo and Alentejana breeds cattle males [2].

We have used maximum likelihood theory to estimate the parameters. However, for cattle data, it is often not feasible to obtain animal's observations at equally spaced ages nor even at the same ages for different animals and there is typically a small number of observations at older ages. For these reasons, maximum likelihood estimates can be quite inaccurate, being interesting to consider in the likelihood function a weight function associated to the elapsed times between two consecutive observations of each animal, which results in the weighted maximum likelihood method. We compare the results obtained from both methods [1].

Additionally, since model parameters may vary from animal to animal and that variability can be partially explained by their genetic differences, we have extended the study to SDE mixed models, where the variation among animals of the parameters α , β or both is assumed to be random. For these three cases, the maximum likelihood estimation method was applied using a Delta method approximation to solve the integrals involved in the maximum likelihood function [3].

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Green Measures for Nonlocal Diffusion Equations

José Luis Silva, Yuri Kondratiev

Abstract

Let $X(t), t \geq 0$ be a time homogeneous Markov process in \mathbb{R}^d starting from the point $x \in \mathbb{R}^d$. For a function $f : \mathbb{R}^d \rightarrow \mathbb{R}$ we consider the following heuristic object

$$V(f, x) = \int_0^\infty \mathbb{E}^x[f(X(t))] dt.$$

If this quantity exists, then $V(f, x)$ is called the *potential* for the function f . The notions of potentials is well known in probability theory, see e.g., [1, 4]. The existence of the potential $V(f, x)$ is a difficult question and the class of admissible f shall be analyzed for each process X separately. One approach is based on the use of the generator L of the process X . Namely, the potential $V(f, x)$ may be constructed as the solution to the following equation:

$$-LV = f.$$

In analogy with the PDE framework, we would like to have a representation

$$V(f, x) = \int_{\mathbb{R}^d} f(y) \mathcal{G}(x, dy),$$

where $\mathcal{G}(x, dy)$ is a measure on \mathbb{R}^d . This measure is nothing but the fundamental solution to the considered equation and traditionally may be called the Green measure for the operator L .

In this talk we study the existence of Green measures for Markov processes with a nonlocal jump generator without second moment and a suitable condition on its Fourier transform. This talk is based, in particular, on the joint works [2, 3].

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Approximations to the Binomial distribution, its bounds, relative and absolute precision

Jorge Santos, Marília Pires, Russell Alpizar-Jara

Abstract

Approximations to the Binomial by a Gaussian distribution are not always consensus and can be tedious, especially when we are dealing with the probability of a range of values of extreme values of the random variate. In the last century, some research led to the use of cumulative normal distribution tables, as they are easily available. This approximation is best for large samples and evenly symmetric situations. We discuss the 3 usual criteria for the applicability of this method. We show that not only sample size and symmetry are important, but also that the error rates are crucial. Small values of the success probability p become unacceptable when we try to calculate tail probabilities that sometimes have the same order of magnitude as the error. Besides, we show that the most restrictive criterion demands more than 50 trials with a probability of success belonging to the range $[0.1;0.9]$, the criteria based on a variance greater than 5 lead to a parabolic shape criterion and the most liberal ones lead to the intersection of two hyperbolic regions. With the increase of computer capabilities this is not a practical problem, but the main idea is helping to provide some guidelines to this subject that is pervasive for recommendations to usual introductory probability and statistics courses.

Keywords: binomial, continuity correction, normal approximation.

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Impulsive coupled systems with functional boundary conditions

Feliz Minhós, Rui Carapinha

Abstract

In this paper we consider a first-order coupled impulsive system of equations with functional boundary conditions, subject to the generalized impulsive effects. It is pointed out that this problem generalizes the classical boundary assumptions, allowing two-point or multipoint conditions, nonlocal and integro-differential ones or global arguments, as maxima or minima, among others. Our method is based on lower and upper solutions technique together with the fixed point theory.

The main theorem is applied to a SIRS model where, to the best of our knowledge, for the first time it includes impulsive effects combined with global, local, and the asymptotic behavior of the unknown functions.

2020 Mathematics Subject Classification: 34B37, 34B15, 92D30.

Keywords: Impulsive problems; upper and lower solutions; fixed point theory, SIRS model.

On the Liapunov convexity theorem under pointwise constraints

Clara Carlota, António Ornelas

Abstract

We present a result that generalizes the Liapunov convexity theorem for vectorial control systems driven by a linear ODE of first-order. Our generalization is in the sense of allowing one to impose a pointwise state-constraint.

Constraint programming approach to multiobjective forest management with adjacency restrictions

Eduardo Eloy, Vladimir Bushenkov, Salvador Abreu

Abstract

Forest management is an activity of prime economic and ecological importance. Managed forest areas can span very large regions and their proper management is paramount to an effective development, in terms both of economic and natural resources planning. Managed activity consists of individual and mutually independent policy choices which apply to distinct patches of land (named stands) which, as a whole, make up the forest area. A forest management plan typically spans a period on the order of a century and is normally geared towards the optimisation of economic metrics (e.g. total wood yield)

In this talk we present a method which uses declarative methods to formalise and solve a long-term forest management problem. We do so based on a state-of-the-art constraint programming system, which we extend to more naturally express concepts related to the core problem.

Artificial Stress Diffusion in Numerical Simulations of Viscoelastic Fluids Flows

Marília Pires, Tomás Bodnár

Abstract

Numerical simulations of viscoelastic fluid flows continue to be a very challenging problem for high values of Weissenberg (We). In order to stabilize the simulations for high values of the parameter We , many researchers add artificial diffusion to the model. In this talk, we present the artificial tensor diffusion of stresses that proved to be very important in stabilizing the simulations for this type of flow. Several variants of tensor artificial diffusion are presented, focusing on practical aspects of its implementation and use.

Dissipation

Joaquim Correia

Abstract

This a talk on Modelling with PDEs (Partial Differential Equations). ‘Approximations’ and ‘regularisations’ are shortly discussed from the point of view of Applied Mathematics, the role of ‘finite speed of propagation’ is stressed.

Period incrementing and Milnor attractors for non autonomous families of at top tent maps

Luís Silva

Abstract

Period adding and period incrementing structures in the bifurcation scenarios of autonomous piecewise-smooth dynamical systems have been described in several works in the last few years, see [2] and references there in. It was observed in [1], that the introduction of a constant plateau in the map leads naturally to the appearance of Milnor attractors, This happens for parameters that may be described as accumulation points of parameters related with period incrementing sequences. In this work we consider families of nonautonomous dynamical systems generated by the sequential iteration defined by a binary sequence s , of two at top tent maps. We describe period incrementing structures in the bifurcation scenarios and show how Milnor attractors emerge as accumulations of period incrementing. We study the dependence of these phenomena on the iteration pattern s

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A Multiscale Network with Percolation Model to Describe the Spreading of Forest Fires

Sara Perestrelo, Clara Grácio, Nuno Ribeiro, Luís Lopes

Abstract

Fire behaves according to three interacting physical factors: fuel availability (morphological and physiological characteristics of vegetation), weather (wind speed and direction, temperature, and relative humidity) and terrain (slope and aspect) [11, 14] – FWT conditions. Fire models such as Rothermel’s [15] predict fire’s local behaviour using fuel model parameters as input. Fuel models, such as [1, 18] are sets of parameters that describe the characteristics that classify certain fuel types.

The field of percolation has played an important role in developing models and strategies to model fire spreading. Works [6, 8, 9] are examples of percolation frameworks to predict fire spreading, based on FWT conditions, using cellular automata (CA).

One of the existing strategies and very important structure in mitigating fire spreading is the implementation of fire-breaks [16]. These are gaps in the available combustible material that prevent the fire from advancing. In [16, 17], the authors use CA modelling to identify efficient fuel break partitions for fire containment and study the efficiency of various centrality statistics, considering GIS meteorological and landscape information data.

Laboratory experiments can be designed to simulate a small-scale fire, but with limitations. An interesting study [2] compares theory results with laboratory simulations, using matchsticks. The theory predicts that at critical percolation a fire front decelerates, whereas experiments indicate acceleration. This discrepancy shows that percolation theory models of forest-fire propagation using simple site percolation are unlikely to be accurate. On the opposite side, large-scale experiments have their limitations in terms of reproducibility as well. Still, percolation modelling of fire-spread is of considerable importance because it describes the transition regime between extinction (spanning fires) and uncontrolled spread (penetrating fires) [2, 7].

To overcome scale limitations, some examples using Multilayer Networks [5, 3, 13] and cellular automata with different approaches have come up with important findings. Work [4], applies multiplex networks to model fire propagation, simulating a 3-layer of possible fire development: ground, surface, and crown, where each node of the multilayer represents a group of trees. At a larger (landscape) scale, work [12] presents a network-of-networks structure, where the nodes are local land patches, with their own spreading dynamics each and, as such, presenting different spreading times.

Our work follows the line of research of the fields of percolation and complex networks. First, we define the local scale as the range in which is possible to delimit a land patch with a measurable set of characteristics, and landscape scale as the scale at which each patch of land is the element of study.

Following the works [12, 13], we present a 2-scale network structure applied to the region of Serra de Ossa, in Portugal. The nodes of the landscape network correspond to territory division in irregular-shaped polygons, based on land characteristics. Within each polygon, SIR simulations occur on a CA, whose cells constitute the local network of the corresponding polygon. Using a classical percolation algorithm, our results for the percolation threshold, p_c 0.407, are consistent with the literature [7, 10]. We then introduce a neighbourhood of warm trees, which change this value to p_c 0.725. The landscape network is then parametrized with p_c values.

The main goal is to find an efficient fire-break structure that mitigates the spreading, to complement the efforts of civil protection forces.. Given the application geography of this model, the complexity of the problem is limited by restricting FWT conditions to those specific to that area. Still, the spatial extent to which our model can be applied strongly depends on previous land monitoring, which implies a demand for the inclusion of other areas of expertise.

As future work, after calibrating the spreading network dynamics, we intend to introduce classifications of autochthonous biomes. Afterwards, vectorial influence inherent to meteorological and orographic factors are to be considered in the simulations in addition to the existing model.

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Flipped classroom as a mathematics learning space for students

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Abstract

In this communication the case study of the main pedagogical technique applied to a Mathematics course unit of a higher educational programme of technology, called flipped classroom [1], will be presented. This technique, in conjunction with others, in the context of the student-centered approach, enhances the significant learning of the course unit's syllabus. The typical passive attitude of the students in the classroom is reversed through learning activities in collaborative working group and knowledge levelling. These activities make it possible to achieve the learning objectives, as well as reinforce or develop the transversal skills of employment, such as autonomy, adaptability, cooperation, constructive criticism and time management. This pedagogical technique was applied in order to work on the specific learning objectives for each class in the context of mathematical content, optimizing the student's academic work time: outside of classes students use virtual resources (texts, videos and other interactive resources) provided by the teacher to learn and deepen the contents and to carry out training tests; during classes students consolidate the knowledge acquired through knowledge levelling activities and perform summative assessments. The teacher assumes the role of facilitator of the entire learning process, guiding students' training by managing the performance of suitable exercises, group activities, by clarifying doubts and by evaluations inside and outside the classes, using the support of an IT platform. The outcomes of the application of this pedagogical technique resulted in a reduction of the dropout rate and an increase in the course unit's success rate, compared to the two previous academic years in which the teacher-centered approach was used. This case study was presented at the 4th International Conference of the Portuguese Society for Engineering Education last year [2].

Keywords: flipped classroom, student-centered approach, active learning, engineering education.

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