

# Constructal theory of flow architecture of the lungs

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Here we explain the reasons why we have a bronchial tree with 23 levels of bifurcation. Based on Bejan's Constructal Principle we found that the best oxygen access to the alveolar tissues as well as carbon dioxide removal is provided by a flow structure composed of ducts with 23 levels of bifurcation (bronchial tree) that ends with spaces (alveolar sacs) from where oxygen diffuses into the tissues. The model delivers the dimensions of the alveolar sac, the total length of the airways, the total alveolar surface area, and the total resistance to oxygen transport in the respiratory tree. A constructal law also emerges: the length defined by the ratio of the square of the airway diameter to its length is constant for all individuals of the same species and is related to the characteristics of the space allocated to the respiratory process and determines univocally the branching level of the respiratory tree. © 2004 American Association of Physicists in Medicine.  
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## NOMENCLATURE

$A$	alveolar area (m <sup>2</sup> )
$d$	diameter of the alveolus (m)
$D$	diffusion coefficient (m <sup>2</sup> s <sup>-1</sup> )
$L$	length from the entrance of the trachea to alveolus (m)
$\dot{m}$	mass flow rate (kg s <sup>-1</sup> )
$M$	molar mass (kg mol <sup>-1</sup> )
$N$	total number of bifurcations
$P$	pressure (Pa)
$Q$	tidal airflow (kg s <sup>-1</sup> )
$r$	resistance (J s kg <sup>-2</sup> )
$R$	total resistance (J s kg <sup>-2</sup> )
$R_g$	specific gas constant (J kg <sup>-1</sup> K <sup>-1</sup> )
$s$	specific entropy (J kg <sup>-1</sup> K <sup>-1</sup> )
$S$	rate of oxygen consumption (kg s <sup>-1</sup> )
$t$	time (s)
$T$	temperature (K)
$u$	specific internal energy (J kg <sup>-1</sup> )
$U$	velocity (m s <sup>-1</sup> )
$V$	volume (m <sup>3</sup> )
$\delta$	distance from the entrance to the surface of the

	alveolus (m)
$\varepsilon$	mechanical energy per unit mass (J kg <sup>-1</sup> )
$\phi$	relative gas concentration
$\eta$	dynamic viscosity (Ns/m <sup>2</sup> )
$\lambda$	characteristic length [Eq. (20)] (m)
$\mu$	chemical potential (J kg <sup>-1</sup> )
$\nu$	kinematic viscosity (m <sup>2</sup> s)
$\rho$	density (kg m <sup>-3</sup> )

## Subscripts

$a$	relative to alveolus
air	relative to air
$B$	relative to the bronchial tree
$bn$	relative to the $n$ th bifurcation
$cd$	relative to carbon dioxide
$cn$	relative to channel of order $n$
diff	diffusive
$i$	running index
$n$	order of bifurcation (0 for trachea)
$N$	relative to the last bifurcation
opt	relative to the optimal value
ox	relative to oxygen

## I. INTRODUCTION

The Constructal Principle that has been originally formulated by Adrian Bejan states that every system with internal flows develops the flow architecture that maximizes the heat and mass flow access under the constraints posed to the flow. In all classes of flow systems (animate, inanimate, engineered) the generation of flow architecture emerges as a universal

phenomenon. This has been shown in a number of articles by Bejan and is summarized in a recent book.<sup>1</sup>

By using the Constructal Principle, Bejan has addressed the rhythm of respiration in animals in relation with the body size and found that the breathing time increases with the animal body size rose to a power of 1/4, which is in good agreement with the biological observations.<sup>1</sup> A number of

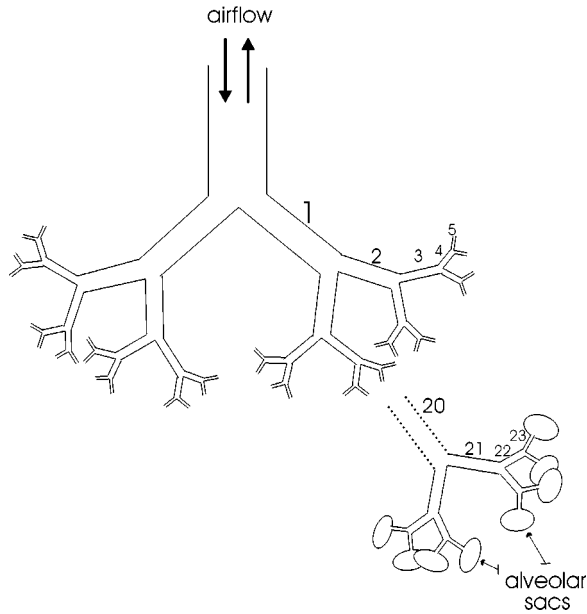


FIG. 1. Model of the respiratory tree with trachea, 23 bronchial bifurcations and alveolar sacs.

other recent studies have focused either on the characteristics of the airflow and gas diffusion within the lungs<sup>2–6</sup> and the form of the arterial bifurcations<sup>7</sup> or statistical description of the respiratory tree.<sup>8,9</sup>

In this work we focus on the structure of the pulmonary airflow tree. The respiratory system is basically a fluid tree that starts at the trachea and bifurcates 23 times before reaching the alveolar sacs.<sup>4,10</sup> The reason for the existence of just 23 bifurcations in the respiratory tree (Fig. 1) has remained unexplained in the literature. Has this special flow architecture been developed by chance or does it represent the optimum structure for the lung's purpose, which is the oxygenation of the blood? The view that the Constructal Principle, which has been originally developed for engineered systems, holds also for living systems, will guide us in finding the best airflow architecture for the respiratory system.

## II. A FLUID TREE WITH PURPOSE

The oxygenation of blood takes place in the tissues that shape the surface of the alveolar sacs. High alveolar surface promotes better oxygenation, but requires increased access to the external air. In fact, if the access to the external air faces high flow resistance the rate of oxygen diffusion into the blood is lowered due to the poor oxygen concentration in the air within the alveolar sacs.

According to the Constructal Principle a fluid tree that performs the oxygenation of blood and removal of carbon dioxide at the lowest flow resistance should exist under the constraints posed by the space allocated to the respiratory process. This fluid tree should be able to promote the easiest access to the external air. Two possibilities exist for accomplishing this purpose: (I) a duct system that ends with an alveolar volume from which the oxygen diffuses to the tissues, where it meets the blood, and in which the carbon

dioxide diffuses after being released from the blood, or (II) a unique volume open to the external air, in which the oxygen reaches the blood in the tissues, and removes the carbon dioxide rejected from the blood, only by diffusion through the internal air.

This second possibility is clearly noncompetitive as compared to the first. The access time for a diffusive process between the entrance of the trachea and the alveolar sacs at a distance  $L \sim 5 \times 10^{-1}$  m is  $t_{\text{diff}} = L^2/D \sim 10^4$  s, where  $D \sim 2 \times 10^{-5}$  m<sup>2</sup>/s is the diffusion coefficient for oxygen in air. The access time for duct flow is of order  $t_{\text{flow}} = 32 \eta L^2 / (D_0^2 \Delta P) \sim 1$  s, where  $D_0 \sim 10^{-2}$  m is the trachea diameter,  $\eta \sim 2 \times 10^{-5}$  Ns/m<sup>2</sup> is air dynamic viscosity, and  $\Delta P \sim 1$  Pa is the scale of the average pressure difference. Therefore the channelling of the air from the outside to the alveolar surface enables a better performance of the respiratory process.

However, a cavity (or alveolar sac) at the end of the channelling tree must exist, as the oxygenation of the blood occurs by diffusion from air into the tissues. Oxygen uptake is proportional to the alveolar surface that, in turn, is proportional to the number of bronchioles corresponding to the final level of bifurcation, which is  $2^N$ ,  $N$  being the number of bifurcation levels. On the other hand, the higher the number of alveolar sacs, the higher the complexity gets, as well as the flow resistance of the duct network. Therefore, the optimum flow structure must emerge from the minimization of the overall resistance, i.e., the duct resistance plus the diffusive resistance.

## III. BRONCHIAL TREE RESISTANCE AND ALVEOLAR RESISTANCE

Oxygen and carbon dioxide flow within the respiratory tree (bronchial tree plus alveolar sacs) under several driving forces. So as to evaluate and compare the flow resistances we will express the flow rates in terms of a unique potential. Airflow within the bronchial tree is assumed to be laminar, isothermal and incompressible. As this flow is also adiabatic, i.e.,  $\Delta s = 0$ , conservation of total energy per unit mass that is the sum of internal energy,  $u = -P/\rho + Ts + \mu$  (where  $P$  is pressure,  $\rho$  is density,  $T$  is temperature,  $s$  is entropy and  $\mu$  is chemical potential) plus kinetic energy  $\varepsilon$ , along the respiratory tree, implies

$$\Delta \mu = \rho^{-1} \Delta P + \Delta \varepsilon. \quad (1)$$

Oxygen and carbon dioxide are assumed to be in equilibrium with the air that flows within the bronchial tree, which means that all gases in the airflow have the same chemical potential and move as a whole between the entrance of the trachea and the alveolar sacs. In this way, the airflow is driven by the gradient of the chemical potential within the bronchial tree, which in each duct is related to the pressure gradient by Eq. (1) as  $\Delta \mu = \rho^{-1} \Delta P$ . Hence, by considering the bronchial tree as composed of cylindrical channels and assuming Hagen–Poiseuille flow, the airflow rate is determined by

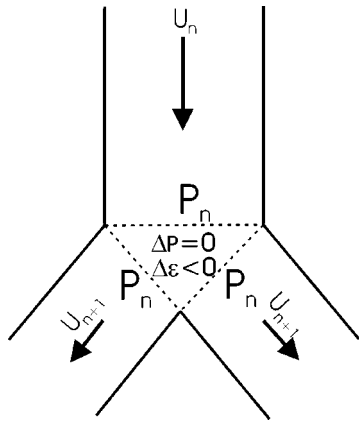


FIG. 2. Hagen–Poiseuille flow in a bifurcation. The resistance to airflow is due to the variation of the average kinetic energy per unit mass, and proportional to the mass flow rate.

$$\dot{m}_n = \frac{\pi \rho D_n^4}{128 \nu L_n} \Delta \mu_{cn}, \tag{2}$$

where  $\dot{m}_n$  and  $\Delta \mu_{cn}$  stand for airflow rate and chemical potential difference between the ends of a channel at the  $n^{\text{th}}$  bifurcation level, respectively, and  $\nu$  is the air kinematic viscosity. For laminar flow the minimum flow resistance at a bifurcation is achieved if the ratio between consecutive duct diameters is<sup>12,14</sup>

$$D_n / D_{n-1} = 2^{-1/3}, \tag{3}$$

and the ratio of the respective lengths,  $L_i$ , is

$$L_n / L_{n-1} = 2^{-1/3}. \tag{4}$$

Equations (3) and (4) represent constructal laws that hold for consecutive channels at a bifurcation. They are robust in the sense that hold for any bifurcation angle<sup>12–14</sup> and express the empirical relation known as Murray’s law. Taking into account Eqs. (3) and (4), the resistance to laminar flow posed by the  $n^{\text{th}}$  bronchial tube is

$$r_{cn} = \frac{\Delta \mu_{cn}}{\dot{m}_n} = 2^n \frac{128 \nu L_0}{\pi \rho D_0^4}, \tag{5}$$

where  $D_0$  and  $L_0$  are the diameter and the length of the first tube in the tree, i.e., the trachea, respectively.

Each bifurcation implies an additional resistance to airflow. In the derivation of Eq. (2), it has been assumed that pressure has no radial variation along each channel. Such a condition implies that in a bifurcation the variation of the chemical potential is entirely due to the variation in the kinetic energy, as shown in Fig. 2. Therefore considering Eq. (1) the airflow rate in a bifurcation may be described by

$$\dot{m}_n = \frac{\Delta \mu_{bn}}{r_{bn}} = - \frac{\Delta \varepsilon_{bn}}{r_{bn}}, \tag{6}$$

where  $\Delta \varepsilon_{bn}$  is the variation of the average kinetic energy per unit mass in the bifurcation. Taking into account the velocity ( $U$ ) profile of cylindrical Hagen–Poiseuille flow, the variation of the kinetic energy per unit mass that flows through a

bifurcation is calculated as  $\Delta \varepsilon = (\rho / \dot{m}_n) (2 \int_{n+1} U^3 dA - \int_n U^3 dA)$ , together with Eqs. (2)–(4), gives the airflow resistance in a bifurcation in the form

$$r_{bn} = \frac{\dot{m}_0}{8 \pi \rho^2 D_0^4} 2^{n/3}, \tag{7}$$

where  $\dot{m}_0$  represents the airflow rate in the trachea. In this way, with the exception of the channels that connect to the alveolar sacs, every other channel may be viewed as having a Hagen–Poiseuille type resistance given by Eq. (5) plus a resistance at the end due to bifurcation given by Eq. (7).

If  $\Delta \mu_n = \Delta \mu_{cn} + \Delta \mu_{bn}$  denotes the total variation of the chemical potential in channels in the  $n^{\text{th}}$  level of bifurcation ( $n=0$ , for the trachea), from Eqs. (5) and (7) and taking into account that in this level there are  $2^n$  bronchial tubes, we obtain the total resistance of the  $n^{\text{th}}$  level as

$$r_n = \frac{128 \nu L_0}{\pi \rho D_0^4} \left( 1 + \frac{\dot{m}_0 2^{-2n/3}}{1024 \rho \nu L_0} \right), \tag{8}$$

then the overall convective resistance of a tree with trachea ( $n=0$ ) plus  $(N-1)$  bifurcation levels is given by

$$R_B = \sum_{n=0}^{N-1} r_n = \frac{128 \nu L_0}{\pi \rho D_0^4} \left[ N + \frac{\dot{m}_0 (1 - 2^{-2N/3})}{379 \pi \rho \nu L_0} \right]. \tag{9}$$

For a normal breathing frequency of 12 times per minute and tidal air of about  $0.5 \text{ dm}^3$ , we conclude that  $\dot{m}_0 (1 - 2^{-2N/3}) / (379 \pi \rho \nu L_0) \ll 1$ , and this term that corresponds to the sum of airflow resistances in the bifurcations may be neglected in Eq. (9).

If  $(\phi_{ox})_0$  and  $\phi_{ox}$  denote the average relative concentration of the oxygen in the air at the entrance of the trachea and at the bronchial tree, respectively, the average oxygen current towards the interior of the bronchial tree is

$$\dot{m}_{ox} = \frac{1}{2} [(\phi_{ox})_0 - \phi_{ox}] \dot{m}_b = \frac{|\Delta \mu_b|}{(R_{ox})_B}, \tag{10}$$

where the subscript ox means oxygen. In Eq. (10) the factor  $1/2$  arises because either inhaling or exhaling takes half of breathing time,  $|\Delta \mu_B| = |\sum_{n=0}^{N-1} \Delta \mu_n|$  is the absolute value of the variation of the chemical potential of the air in the trachea plus the  $(N-1)$  levels of bifurcation, and  $(R_{ox})_B = 2R_B / ((\phi_{ox})_0 - \phi_{ox})$  is the resistance to oxygen transport.

However, no such equilibrium conditions exist between the components of the air within the alveolar sacs, because the chemical potential of oxygen in the alveolar tissues is lower than that in the alveolar air, while the chemical potential of carbon dioxide in the tissue is higher than that in the alveolar air. Therefore oxygen diffuses from the alveolar air into the tissues, while carbon dioxide diffuses in the opposite direction. It is assumed that oxygen diffuses at the  $2^N$  alveolar sacs according to Fick’s law, consequently the total oxygen current to the alveolar sacs, which are considered to be in a spherical shape with diameter  $d$  and total area  $\pi d^2$  (see Fig. 3), is given by

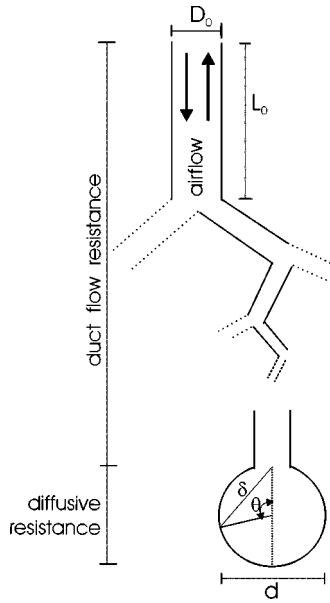


FIG. 3. Model of the respiratory tree with a conductive part (bronchioles) and a diffusive space (alveolar sac).

$$\dot{m}_{\text{ox}} = 2^N \int_{\pi}^0 D_{\text{ox}} \frac{(\Delta\rho_{\text{ox}})_a \pi d^2 \sin\theta d\theta}{2\delta}, \quad (11)$$

where  $D_{\text{ox}}$  is the oxygen diffusivity,  $(\Delta\rho_{\text{ox}})_a$  is the difference between the oxygen concentrations at the entrance of the alveolar sac and the alveolar surface, and  $\delta = d\sqrt{(1 - \cos\theta)/2}$  (see Fig. 3). Taking into account that  $(\Delta\rho_{\text{ox}})_a = \phi_{\text{ox}}\rho(\Delta\mu_{\text{ox}})_a / (R_g)_{\text{ox}}T$ , where  $(R_g)_{\text{ox}} = R/M_{\text{ox}}$  is the gas constant for oxygen and  $\phi_{\text{ox}}$  is the relative concentration of oxygen in the alveolar air, and assuming that the chemical potential of oxygen does not vary over the alveolar surface, integration of the rhs of Eq. (11) yields

$$\dot{m}_{\text{ox}} = 2^N \frac{2\pi d\rho\phi_{\text{ox}}D_{\text{ox}}(\Delta\mu_{\text{ox}})_a}{(R_g)_{\text{ox}}T}. \quad (12)$$

The diameter of the alveolar sac may be determined as the difference between the overall lengths  $L$  of a bronchial tree with infinite bifurcations, which is the limiting length defined by the constructal law, Eq. (4), and that of the actual tree with  $N$  bifurcation levels, i.e.,

$$d = L - \sum_{i=1}^N L_i. \quad (13)$$

The length  $L_N$  of a tree with  $N$  bifurcations may be determined from Eq. (4) as the length of the trachea plus the lengths of the  $N$  consecutive bronchioles is given by the sum of  $N+1$  terms of a geometric series of ratio  $2^{-1/3}$  as

$$\sum_{i=0}^N L_i = \frac{1 - 2^{-(N+1)/3}}{1 - 2^{-1/3}} L_0. \quad (14)$$

Therefore,  $L = \lim_{N \rightarrow \infty} \sum_{i=0}^N L_i$ , namely  $L = 4.85 L_0$ . Equations (13) and (14) enable us to determine the diameter of the alveolar sac as

$$d = 4.85 \times 2^{-(N+1)/3} L_0. \quad (15)$$

In consequence, Eq. (12) may be written as

$$\dot{m}_{\text{ox}} = 7.70\pi L_0 \phi_{\text{ox}} \rho D_{\text{ox}} \frac{2^{2N/3}}{(R_g)_{\text{ox}}T} (\Delta\mu_{\text{ox}})_a. \quad (16)$$

In this way, the resistance of the  $N$ th level of bifurcation that is the sum of the convective resistance of the last  $2^N$  channels, which is given by  $2^{-N}r_{cN}$  [see Eq. (5)], plus the alveolar diffusive resistance given by Eq. (16) [see also Eq. (10)] is

$$(R_{\text{ox}})_N = \frac{128\nu L_0}{\pi[(\phi_{\text{ox}})_0 - \phi_{\text{ox}}]\rho D_0^4} + 0.13(R_g)_{\text{ox}}T \frac{2^{-2N/3}}{\pi\phi_{\text{ox}}\rho L_0 D_{\text{ox}}}. \quad (17)$$

Therefore, if  $\Delta\mu_{\text{ox}} = (\Delta\mu_{\text{ox}})_B + (\Delta\mu_{\text{ox}})_N + (\Delta\mu_{\text{ox}})_a$  is the total difference between the chemical potential of the oxygen in the external air and the oxygen close to the alveolar surface, the total resistance,  $R_{\text{ox}} = \Delta\mu_{\text{ox}}/\dot{m}_{\text{ox}}$ , posed to oxygen as it moves from the external air into the alveolar surface, which is the sum of the resistance  $(R_{\text{ox}})_B$ , given by Eq. (9), with  $(R_{\text{ox}})_N$  given by Eq. (17), reads as

$$R_{\text{ox}} \cong \frac{256\nu L_0}{\pi D_0^4[(\phi_{\text{ox}})_0 - \phi_{\text{ox}}]\rho} (N+1) + \frac{0.13(R_g)_{\text{ox}}T 2^{-2N/3}}{\pi L_0 D_{\text{ox}} \phi_{\text{ox}} \rho}, \quad (18)$$

where the resistances in the bifurcations have been neglected due to the fact that its value is very small as compared to channel resistances. The total resistance is composed of a convective resistance and a diffusive resistance represented by the first and the second terms of the rhs of Eq. (18), respectively.

#### IV. OPTIMIZATION OF THE RESPIRATORY TREE ON THE BASIS OF THE CONSTRUCTAL PRINCIPLE

Following the Constructal Principle, the flow architectures evolve in time in order to maximize the flow access under the constraints posed to the flow. We believe that, during millions of years of human evolution, the oxygen-access performance of the respiratory tree was optimized naturally, through changes in flow architecture.

In Eq. (18), the convective part of the resistance increases as the number of bifurcations increases, while the diffusive resistance decreases. The number of bifurcations is the free parameter that can be optimized in order to maximize the oxygen access to the alveolar surface or, in other words, to minimize the total resistance to oxygen access.

The average value of oxygen relative concentration within the respiratory tree,  $\phi_{\text{ox}}$ , may be evaluated from the alveolar air equation in the form  $((\phi_{\text{ox}})_0 - \phi_{\text{ox}})Q - S = 0$ , where

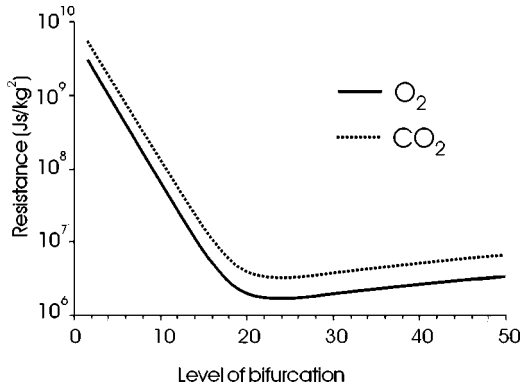


FIG. 4. Total resistance to oxygen and carbon dioxide transport between the entrance of the trachea and the alveolar surface is plotted as function of the level of bifurcation ( $n$ ). The minimum resistance both to oxygen access and carbon dioxide removal corresponds to  $N=23$ .

( $\phi_{\text{ox}})_0 \sim 1/2(\phi_{\text{air}} + \phi_{\text{ox}})$  and  $\phi_{\text{air}}$  are the oxygen relative concentration at the entrance of the trachea, and in the external air, respectively,  $Q$  is the tidal airflow and  $S$  is the rate of oxygen consumption. With  $\phi_{\text{air}}=0.2095$ ,  $Q \sim 6 \text{ dm}^3/\text{min}$  and  $S \sim 0.3 \text{ dm}^3/\text{min}^{5,11}$  we obtain  $\phi_{\text{ox}} \sim 0.1095$ .

For  $L_0$  we take the sum of the larynx and trachea lengths (first duct), which is typically 15 cm, while the trachea diameter,  $D_0$ , is approximately 1.5 cm.<sup>10,11</sup> Air and oxygen properties were taken at 36 °C, namely  $\nu = 1.7 \times 10^{-5} \text{ m}^2/\text{s}$ ,  $D_{\text{ox}} = 2.2 \times 10^{-5} \text{ m}^2/\text{s}$ ,  $(R_g)_{\text{ox}} = 259.8 \text{ J}/(\text{kg}\cdot\text{K})$ . The plot of the total resistance of the respiratory tree against the bifurcation level is shown in Fig. 4. It can be seen that the minimum is flat and occurs close to  $N=23$ .

A more accurate value of this minimum is obtained analytically from Eq. (18). The optimum number of bifurcation levels is given by

$$N_{\text{opt}} = 2.164 \ln \left[ \frac{2.35 \times 10^{-4} D_0^4 (R_g)_{\text{ox}} T \left( \frac{(\phi_{\text{ox}})_0}{\phi_{\text{ox}}} - 1 \right)}{L_0^2 \nu D_{\text{ox}}} \right], \quad (19)$$

which yields  $N_{\text{opt}} = 23.4$ . As  $N$  must be an integer, this means that the optimum number should be 23. In view of the simplifications of the model (mainly the geometry of the bronchial tubes which are assumed to be cylindrical and the geometry of the alveolar sacs which are viewed as spheres, this result is in a very good agreement with the literature, which indicates 23 as the number of bifurcations of the human bronchial tree.<sup>4,10</sup>

The respiratory tree can also be optimized for carbon dioxide removal from the alveolar sacs. In this case the correspondent equation to  $N_{\text{opt}}$  is Eq. (18) with the rhs multiplied by  $-1$  and the correspondent values of the diffusion coefficient, which is  $D_{\text{cd}} = 1.9 \times 10^{-5} \text{ m}^2/\text{s}$  for carbon dioxide, the gas constant  $(R_g)_{\text{cd}} = 189 \text{ J kg}^{-1} \text{ K}^{-1}$ , and the value of the average relative concentration of carbon dioxide in the respiratory tree,  $\phi_{\text{cd}} = 0.04$ . In the calculation of  $\phi_{\text{cd}}$  we used  $S = 0.24 \text{ dm}^3/\text{min}$  since the respiratory coefficient is close to 0.8 and  $(\phi_{\text{cd}})_{\text{air}} \sim 0.315 \times 10^{-3}$ . The plot of the resistance to

carbon dioxide removal against bifurcation level is shown in Fig. 4. The minimum resistance, as calculated from Eq. (18), corresponds to  $N_{\text{opt}} = 23.2$ .

We can say that the human respiratory tree, with its 23 bifurcations, is optimized for both oxygen access and carbon dioxide removal. For  $N=23$  the resistance to carbon dioxide removal is  $4.8 \times 10^6 \text{ J s kg}^{-2}$  and higher than the resistance to oxygen access that is  $2.60 \times 10^6 \text{ J s kg}^{-2}$ .

One of the initial assumptions of this model of respiratory tree was that diffusion can be neglected within the bronchial tree where oxygen is transported in the airflow while diffusion is the main way of oxygen transport in the alveolar sacs. By using Eq. (2) and considering tidal volume of  $0.5 \text{ dm}^3$ , breathing frequency of 12 times per minute and trachea diameter of 0.015 m, we calculate the average velocity of the airflow, and therefore of the oxygen current, in the last bronchiole before the alveolar sac, which is of order 6 mm/s. On the other hand, the average velocity of the diffusive current of oxygen in the alveolar sacs is of order  $D_{\text{ox}}/2\pi d \sim 1.3 \text{ mm/s}$ . These results are consistent with the initial assumptions of the model. However, as in this idealized model the velocities of the oxygen for convective and diffusive current simply approach each other, in the real respiratory tree we can expect that in some branches they are of the same order, what indicates the possibility of developing alveoli before the end of the bronchial tree as really happens in the human respiratory tree.<sup>5</sup>

If the number  $N_{\text{opt}} = 23$  is common to mankind, then a constructal law emerges from Eq. (19): “the ratio of the square of the trachea diameter to its length is constant and a length characteristic of mankind,”

$$\lambda = \frac{D_0^2}{L_0} = \text{const} = 1.5 \times 10^{-3} \text{ m}. \quad (20)$$

This number has a special relationship with some special features of the space allocated to the respiratory process, as we show next. From Eqs. (3) and (4) we can estimate the volume occupied by the bronchial tree, which is the sum of the volumes of the 23 bifurcation levels, as  $V_B = 23 \times (\pi/4) D_0^2 L_0$ . The total volume of the alveolar sacs is  $V = 2^{23} (\pi/6) d^3$ . We see that  $V_B/V \ll 1$ , which means that the volume of the lungs practically corresponds to the volume,  $V$ , occupied by the alveolar sacs. The internal area of the alveolar sacs is  $A = 2^{23} \times \pi d^2$ , and therefore  $A/V = 6/d$ . By using Eqs. (14), (15), which lead to  $L = 2^{(N+1)/3} d$ , together with Eq. (18), we obtain the following relationship:

$$\frac{D_0^2}{L_0} = 8.63 \frac{AL}{V} \left\{ \frac{\nu D_{\text{ox}} \phi_{\text{ox}}}{(R_g)_{\text{ox}} T [(\phi_{\text{ox}})_0 - \phi_{\text{ox}}]} \right\}^{1/2}. \quad (21)$$

The nondimensional number  $AL/V$  determines the characteristic length  $\lambda = D_0^2/L_0$ , which determines the number of bifurcations of the respiratory tree by Eq. (19). This constructal law is formulated as follows: “The alveolar area required for gas exchange,  $A$ , the volume allocated to the respiratory system,  $V$ , and the length of the respiratory tree,  $L$ , which

are constraints posed to the respiratory process determine univocally the structure of the lungs, namely the bifurcation level of the bronchial tree.”

From Eq. (14), we obtain  $L = 4.85 L_0 = 0.73$  m for the total length of the respiratory tree and from Eq. (21) we obtain  $d = 6 V/A = 2.8 \times 10^{-3}$  m. Therefore we have the alveolar surface area as  $A = 2^{23} \pi d^2 = 215.5$  m<sup>2</sup> and the total alveolar volume  $V = 2^{23} (\pi/6) d^3 = 102.7$  dm<sup>3</sup>. The alveolar surface area  $A$  is not far from the values found in literature that fall in the range 100–150 m<sup>2</sup>. However, the value found for the total alveolar volume is much higher than the average lung capacity ( $\sim 7.5$  dm<sup>3</sup>). This may arise from the lung’s volume being calculated as the alveolar sacs were fully inflated. Nevertheless, the value found for total alveolar volume suggests that the dimension of the alveolar sacs was somehow overestimated.

## V. CONCLUSIONS

The Constructal Principle that has been successfully employed in engineered systems also proved to be a fundamental tool for the study of flow structures like the respiratory tree. The best oxygen access to the tissues where it reaches the blood is performed by a flow structure composed of ducts with 23 levels of bifurcation. The same structure has been shown to being optimized for carbon dioxide removal as well. At the end of the smallest duct, spaces exist (alveolar sacs) from which the oxygen diffuses to the tissues and in which the carbon dioxide that is removed from the tissues diffuses before reaching the bronchial tree that transports it to the exterior air. The optimized number of bifurcation levels matches the 23 levels that the physiology literature indicates for the human bronchial tree.

In addition, the optimization also predicts the dimension of the alveolar sac, the total alveolar surface area, the total alveolar volume, and the total length of the airways. These values agree, at least in an order of magnitude sense with the values found in the physiology literature. Furthermore, it was shown that the length  $\lambda$  (defined as the ratio between the square of the first airway diameter and its length) is constant for every individual of the same species and related to the characteristics of the space allocated for the respiratory process. This number is univocally determined by a nondimensional number,  $AL/V$ , which involves the characteristics of

the space allocated to the respiratory system, namely the total alveolar area,  $A$ , the total volume  $V$ , and the total length of the airways,  $L$ .

In general, we conclude, from Eq. (19), that for every species whose respiratory tree is optimized the same rule must hold, and exhibit the respective characteristic length,  $\lambda$ .

The application of the Constructal Principle to the generation of the optimal configuration of the respiratory tree was based on the view that Nature has optimized the living flow structures, in time. The work described in this paper supports this view. In this way the Constructal Principle promises to serve as a basis for optimizing and anticipating other living flow structures.

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<sup>1</sup>A. Bejan, *Shape and Structure, from Engineering to Nature* (Cambridge University Press, Cambridge, 2000).

<sup>2</sup>J. S. Andrade *et al.*, “Asymmetric flow in symmetric branched structures,” *Phys. Rev. Lett.* **81**, 926–929 (1998).

<sup>3</sup>B. Mauroy, M. Filoche, J. S. Andrade, and B. Sapoval, “Interplay between geometry and flow distribution in an airway tree,” *Phys. Rev. Lett.* **90**, 926–929 (2003).

<sup>4</sup>V. V. Kulish, J. L. Lage, C. C. W. Hsia, and R. L. Jonhson, “A porous medium model of alveolar gas diffusion,” *J. Porous Media* **2**, 263–275 (2002).

<sup>5</sup>V. V. Kulish and J. L. Lage Kulish, “Fundamentals of alveolar diffusion: a new modeling approach,” *Automedica* **20**, 225–268 (2001).

<sup>6</sup>V. V. Kulish, J. L. Lage, C. C. W. Hsia, and R. L. Jonhson, “Three-dimensional, unsteady simulation of alveolar respiration,” *J. Biomed. Eng.* **124**, 609–616 (2002).

<sup>7</sup>K. L. Karau, G. S. Krenz, and C. A. Dawson, “Branching exponent heterogeneity and wall shear stress distribution in vascular trees,” *Am. J. Physiol.* **280**, 1256–1263 (2001).

<sup>8</sup>C. G. Phillips, S. R. Kaye, and R. C. Schroter, “A diameter-based reconstruction of the branching pattern of the human bronchial tree. I. Description and application,” *Respir. Physiol.* **98**, 193–217 (1994).

<sup>9</sup>C. G. Phillips, and S. R. Kaye, “Diameter-based analysis of the branching geometry of four mammalian bronchial trees,” *Respir. Physiol.* **102**, 303–316 (1995).

<sup>10</sup>H. Brad *et al.*, “Bronchial anatomy,” *Virtual Hospital*, <http://www.vh.org/>, University of Iowa (2003).

<sup>11</sup>F. Roberts, “Update in Anaesthesia,” *Respir. Physiol.* **12**, 1–3 (2000).

<sup>12</sup>S. Lorente, W. Wechsato, and A. Bejan, “Three-shaped flow structures designed by minimizing path lengths,” *Int. J. Heat Mass Transfer* **45**, 3299–3312 (2002).

<sup>13</sup>A. Bejan, L. A. O. Rocha, and S. Lorente, “Thermodynamic optimization of geometry: T- and Y-shaped constructs of fluid streams,” *S. Int. J. Thermal Sci.* **39**, 949–960 (2000).

<sup>14</sup>A. Bejan, I. Dincer, S. Lorente, A. F. Miguel, and A. H. Reis, *Porous and Complex Flow Structures in Modern Technologies* (Springer-Verlag, New York, 2004), Chap. 4.